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RESEARCH

Parameter Design of Materials Processing in Term of Probabilistic Multi-objective Optimization

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ABSTRACT

Parameter design of material processing is quite significant to provide a safeguard to the quality of product comprehensively in condition of clean production especially. In this paper, an appropriate approach of parameter design of materials processing is proposed in term of probabilistic multi-objective optimization (PMOO). The approach has the characteristic of concurrent optimization of multiple objectives in spirit of probability theory inherently; furthermore the "sequential number-theoretic optimization (SNTO)" is employed to conduct the discretization of successive deep optimization. Besides, the optimal design of materials processing is completed by conducting the assessment of total preferable probability for each scheme. Subsequently, parameter design problems of grinding processes of H7007C bearing inner ring with energy saving and emission reduction, and processing optimization of aluminum alloy AA 6082 blank hot stamping, are taken as examples to illuminate the procedure of the approach, respectively. The results show the rationality of the approach. It has a bright prospect in parameter design of production optimization in the future.

KEYWORDS

Design; material processing; sustainable technology, probabilistic method; multi - objective optimization.

INTRODUCTION

More attention has been continuously paid to the development of artificial intelligence and manufacturing technique, clean production and environmental protection issues in recent years. Various multi - objective optimization methods have been put forward to provide an optimal solution for parameter design in materials processing, which attempts to supply appropriate parameter design for material processing systematically.

Till now, Vlšekriterijumska Optimizacija I KOmpromisno Resenje (VIKOR), Analytical Hierarchy Process (AHP), Technique of ranking Preferences by Similarity to the Ideal Solution (TOPSIS), and Multi-Objective Optimization on the basis of Ratio Analysis (MOORA), etc., are commonly employed methods [**1-5**].

Actually, the essence of multi - objective optimization (MOO) is the "concurrent Optimization of Multiple Objectives" inevitably. But the previous approaches regarding to MOO took the "additive" algorithm as the actual operation and normalization of the evaluation indexes in parameterization, and introductions of weight factors, etc. [**1-5**].

In fact, the "additive" algorithm of multiple evaluation indexes has its specific feature of "union" in set theory and probability theory, which is in fact not consistent with the essence of "cobcurrent optimization of multiple indexes" instead [**6**]. Moreover, in the spirit of probability theory, "concurrent optimization of multiple objectives" should take the form of "joint probability" of the multiple objectives appropriately.

Additionally, since the introduction of subjective factors in previous approaches, the corresponding algorithms could only be thought as a semi - quantitative one in some sense. Besides, the selection of normalized factor in the normalization process is puzzled, different selection of normalized factor could usually lead to quite different consequences [6].

Therefore, the proper description for "concurrent optimization of multiple objective" in quantitative manner is still on its way.

In view of this status, Zheng *et al* recently took each objective of the concurrent optimization problem of the multiple objectives as an independent event in the spirit of probability theory [6], and the entire problem of the multi - objective optimization as a "joint event" of all independently individual events together. Thereafter, the overall/total probability of "joint event" is the multiplication of each independently individual event in whole thing. Additionally, each independently individual event can be assessed in accordance with its role or preference degree in the evaluation quantitatively with "partial preferable probability", thus the concurrent optimization of multiple objectives can be reasonably characterized by total / overall preferable probability of the "joint event". This methodology is with the advantage of taking concurrent optimizations of these multiple objectives in light of probability theory.

In this article, an appropriate approach of parameter design of materials processing is proposed in term of the probabilistic multi-objective optimization (PMOO). Furthermore, the hybrid of PMOO with sequential number-theoretic optimization (SNTO) is employed to perform the deep optimization subsequently. In the approach, the role of PMOO is to perform the transformation of the optimization problem with multiple objectives into a mono objective one in the spirit of probability theory, while the "good lattice point" and "uniform design method" are employed in the sequential optimization process to complete the discretization in the deep optimization. By completing the evaluation of overall preferable probability of each scheme, the optimal design of material processing is accomplished rationally.

Moreover, two examples including design problems of grinding processes of H7007C bearing inner ring with energy saving and emission reduction, and optimization of hot stamping processing of aluminum alloy AA 6082 blank, are employed to illuminate the procedure of the approach in details.

COMBINATION OF PMOO WITH SEQUENTIAL UNIFORM DESIGN

The combination of probabilistic multi – objective optimization with sequential number-theoretic optimization in term of sequential uniform design is used to regulate an appropriate design for material processing with multiple objectives according to following procedures.

Main idea of preferable probability

The main idea of preferable probability was introduced in PMOO to describe the preference degree of an attribute (objective) in the assessment [6]. In the methodology, the attributes (objectives) are preliminarily categorized into two types, i.e., both beneficial and unbeneficial types. Furthermore, the quantitative assessment of the partial preferable probability of each type of attribute index is established. Besides, the concurrent optimization of multiple objectives was taken as the product of "partial preferable probability" of all independently individual objective events, which forms the overall / total preferable probability of an alternative. The overall / total preferable probability is unique / overall index of "joint event" of all possible objectives of the candidate alternative, which is the decisive indicator of the candidate alternative in the optimization. Thus, the multi – objective optimization problem transfers into a mono objective one.

Evaluation of preferable probability

As the preferable probability is a characterization of the preference degree of the utility value of an attribute quantitatively, and the value of an attribute reflects the feature of this attribute in one aspect, so the preferable probability could be reasonably related to the value of the utility value of the attribute. For simplicity, the value of partial preferable probability of attribute in a beneficial type (the bigger the better type) is set to be in direct proportional to the utility value of the corresponding attribute [6],

$$P_{ij} \propto X_{ij},\ P_{ij} = \alpha_j X_{ij},\ i = 1, 2, \ldots, m;\ j = 1, 2, \ldots, n. \tag{1}$$

In Eq. (1), X_{ij} is the utility value of the j-th attribute of i-th candidate scheme; P_{ij} is the partial preferable probability of beneficial attribute X_{ij}; α_j is the normalized coefficient of the j-th attribute in beneficial type; m is the total number of candidate scheme; n is the number of attributes of candidate scheme.

Equivalently, the partial preferable probability of a unbeneficial type of attribute could be written as [6],

$$P_{ij} \propto -X_{ij},\ P_{ij} = \beta_j (X_{jmax} + X_{jmin} - X_{ij}),\ i = 1, 2, \ldots, m;\ j = 1, 2, \ldots, n. \tag{2}$$

where β_j is the normalized coefficient of the j-th attribute in unbeneficial type, X_{jmax} and X_{jmin} represent the maximum and minimum values of the utility value in the group of j-th attribute, respectively.

Furthermore, the normalized coefficients of α_j and β_j can be obtained according to the normalization rule in probability theory, i.e., $\sum_{i=1}^{n} P_{ij} = 1$,

$$\alpha_j = \frac{1}{n\overline{X_j}},\ \beta_j = \frac{1}{n(X_{j\max} + X_{j\min} - n\overline{X_j})} \tag{3}$$

where $\overline{X_j}$ indicates the arithmetic mean value of the utility value in the group of j-th attribute [6].

Moreover, the overall / total preferable probability of i-th alternative scheme is the product of its all-possible partial preferable probabilities P_{ij} in term of joint probability algorithm in probability theory [**6**],

$$P_i = P_{i1} \cdot P_{i2} \cdots P_{im} = \prod_{j=1}^{m} P_{ij} \,. \tag{4}$$

The overall / total preferable probability of i-th alternative scheme is its unique and decisive indicator in the optimization. Finally, the optimal scheme is with the highest value of overall / total preferable probability among all schemes.

Hybrid of PMOO with SNTO

The hybrid of PMOO with sequential uniform design aims to complete the succeeding optimal design.

The sequential uniform design was proposed as a sequential number-theoretic optimization (SNTO) for uniform design with NT-nets in seeking maximum value by Fang and Wang [**7**], which can be integrated with PMOO to perform the successive optimal design deeply [**6**].

The processing of the successively deep optimal design is to contract the seeking range of independent variables so as to conduct the discretization for the succeeding evaluations step by step [**6,7**], which is demonstrated in **Fig. 1**.

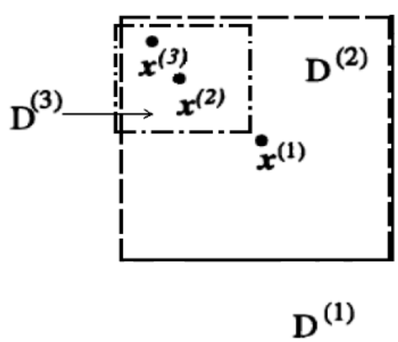

Fig. 1. Demonstration of the procedure of SNTO

The uniform table is employed to spread sampling points within the domain, which is thus used to conduct the succeeding assessments for total / overall preferable probability of each sampling point (alternative scheme). The process is repeatedly conducted till the relative error of total / overall preferable probability being smaller than a pre-assigned value [**7**].

UTILIZATION EXAMPLES OF THE APPROACH IN PARAMETER DESIGN OF MATERIALS PROCESSING

Parameter design of grinding process with energy saving and emission reduction

Lv *et. al.* conducted parameter design of grinding processes of H7007C bearing inner ring with energy saving and emission reduction by combination of orthogonal experimental design together with Matlab [**8**]. Genetic algorithm was employed in their multi - objective optimization. Here, the same problem is restudied by using PMOO.

The Taguchi design $L_{16}(4^5)$ was employed [**8**], the input variables include the grinding wheel linear speed v, workpiece speed ω, and feed rate s. The roughness Ra, total energy consumption E, carbon emissions C, grinding time T were taken as objectives in the study [**8**]. The consequences of their simulation are cited in **Table 1**, their simulation was performed by combination of orthogonal experimental design together with Matlab. The designed parameters of input variables are cited in **Table 2** [**8**]. According to the essence of this optimization, all above objectives are unbeneficial type of index [**8**]. **Table 3** shows the assessment results in term of PMOO, which shows that scheme No. 13 has the biggest value of total preferable probability at the first glance. **Table 4** shows the result of range analysis of optimal design of the grinding processes with energy saving and emission reduction. The result of range analysis indicates that the optimal configuration for this problem is $v_4\omega_1s_3$, which is exactly the same as the scheme No. 13. **Table 5** shows the comparison of the optimized parameters with those of industry and Lv *et. al.*, which indicates a superior of the present results to the industry and Lv *et. al.* obviously [**8**].

Table 1. Simulation results of the grinding processes.

No.	Grinding wheel speed, v (m/s)	Work piece speed, ω (r/min)	Feed rate, s (mm/min)	Rough ness Ra, (μm)	Total energy con-sump-tion E, (W·h)	Carbon emi-ssion, C (g)	Grin ding time, T (s)
1	70.65	150	0.05	0.023	42.69	34.92	60
2	70.65	200	0.1	0.065	23.41	25.26	30
3	70.65	250	0.15	0.069	17.17	22.13	20
4	70.65	300	0.2	0.096	13.77	20.43	15
5	78.5	150	0.1	0.041	25.67	26.39	30
6	78.5	200	0.05	0.02	45.79	36.47	60
7	78.5	250	0.2	0.08	14.77	20.92	15
8	78.5	300	0.15	0.055	20.85	23.97	20
9	86.35	150	0.15	0.052	23.11	25.1	20
10	86.35	200	0.2	0.063	20.66	23.86	15
11	86.35	250	0.05	0.037	46.97	37.06	60
12	86.35	300	0.1	0.042	25.74	26.42	30
13	94.2	150	0.2	0.035	16.1	21.59	15
14	94.2	200	0.15	0.049	19.4	23.24	20
15	94.2	250	0.1	0.041	34.12	30.61	30
16	94.2	300	0.05	0.035	57.04	42.09	60

Table 2. Designed parameters of input variables.

Level	Input variable		
	v (m/s)	*ω (r/min)*	*s (mm/min)*
1	70.65	150	0.05
2	78.50	200	0.10
3	86.35	250	0.15
4	94.20	300	0.2

Table 3. Assessments of design of grinding processing with probabilistic multi – objective optimization for energy saving and emission reduction.

	Preferable probability					Rank
No.	P_{Ra}	P_E	P_C	P_T	$P_t \times 10^5$	
1	0.0883	0.0410	0.0493	0.0214	0.3826	13
2	0.0484	0.0691	0.0666	0.0643	1.4324	10
3	0.0446	0.0782	0.0721	0.0786	1.9792	6
4	0.0190	0.0832	0.0752	0.0857	1.0181	12
5	0.0712	0.0658	0.0645	0.0643	1.9452	7
6	0.0912	0.0365	0.0465	0.0214	0.3317	14
7	0.0342	0.0817	0.0743	0.0857	1.7795	9
8	0.0579	0.0729	0.0689	0.0786	2.2835	3
9	0.0608	0.0696	0.0668	0.0786	2.2204	4
10	0.0503	0.0731	0.0691	0.0857	2.1788	5
11	0.0750	0.0348	0.0455	0.0214	0.2542	15
12	0.0703	0.0657	0.0645	0.0642	1.9147	8
13	0.0769	0.0798	0.0731	0.0857	3.8460	1
14	0.0636	0.0750	0.0702	0.0786	2.6298	2
15	0.0712	0.0535	0.0570	0.0643	1.3964	11
16	0.0769	0.0201	0.0365	0.0214	0.1208	16

Table 4. Result of range analysis of grinding processing with probabilistic multi – objective optimization for energy saving and emission reduction.

	Variable		
Level	***v***	***ω***	***s***
1	1.2031	2.0985	0.2723
2	1.5850	1.6432	1.6722
3	1.6420	1.3523	2.2782
4	1.9982	1.3343	2.2056
Range	0.7952	0.7643	2.0059
Order	2	3	1
Optimal Conf.	4	1	3

Table 5. Comparison of the optimized parameters with respect to industrial one and Lv.

Source	Optimal response			
	Ra, (μm)	*E (W·h)*	*C (g)*	*T(s)*
Industry	0.052	23.11	25.10	20
From (Lv, *et. al*, 2022)	0.057	18.76	22.95	15
Optimized here	0.035	16.1	21.59	15

Processing optimization of aluminum alloy blank hot stamping

Ma et. al. conducted optimization of hot stamping processing of aluminum alloy AA 6082 blank by using FEM and Pareto optimum algorithm [**9**]. The simulation software was Pamstamp. The size of the Al alloy sheet is *900 mm × 700* mm with central composite experimental design [**9**].

The range of the forming temperature of sheet metal is *400* to *520 °C*, and the edge force ranges from *110* to *350 kN* [**9**].

The maximum thinning rate y_1 and the maximum springback amount y_2 are taken as two optimal objectives; while the forming temperature x_1 and blank holder force x_2 are taken as design variables [**9**]. Here it is restudied.

Obviously, the maximum thinning rate y_1 and the maximum springback amount y_2 all belong to unbeneficial type of indexes.

The regressed expressions of y_1 and y_2 *vs* x_1 and x_2 from their simulation were formulated as [**8**],

$$y_1 = 38.77802 - 0.12939x_1 + 0.017669x_2 + 1.3075\times10^{-4} x_1 x_2 + 1.41432\times10^{-4}x_1^2 + 5.99871\times10^{-5}x_2^2 \quad (5)$$

$$y_2 = 61.01602 - 0.19256 x_1 - 0.036866 x_2 + 7.28289\times10^{-5} x_1x_2 + 1.54937\times10^{-4}x_1^2 + 1.06841\times10^{-5}x_2^2 \quad (6)$$

Here, the uniform table $U_{37}(37^{12})$ is used to conduct the initial discretization of this problem, which is with two variables shown in **Table 6** [**10**]. In addition, the assessment consequences of the partial preferable probabilities of y_1 and y_2 and the total preferable probabilities for all discrete points are presented in **Table 6** as well. The values of y_1 and y_2 are obtained by substituting the discretized values of x_1 and x_2 into the expressions of y_1 and y_2, i.e., Eqs. (5) and (6) directly. From Table 6, the highest value of the total preferable probability for this initial design is located at the scheme No. 35 with specific values of $x_1^* = 511.8920°C$, $x_2^* = 126.2160\ kN$, and the corresponding responses are $y_1 = 21.2376\%$, $y_2 = 3.2673\ mm$, respectively.

Table 6. Initial design and assessments of the multi – objective optimization on hot stamping of aluminum alloy blank processing with $U_{37}(37^{12})$.

	Variable		Response		Preferable probability			Rank
No.	x_1, °C	x_2, KN	y_1, %	y_2, mm	P_{y1}	P_{y2}	$P_t \times 10^3$	
1	401.6216	217.0270	27.6818	7.5214	0.0319	0.0159	0.5077	
2	404.8649	327.2973	39.1103	7.1808	0.0223	0.0176	0.3916	
3	408.1081	197.5676	25.9033	7.2414	0.0334	0.0173	0.5771	
4	411.3514	307.8378	37.1656	6.9091	0.0239	0.0189	0.4520	
5	414.5946	178.1081	24.1491	6.9643	0.0348	0.0187	0.6497	
6	417.8378	288.3784	35.2452	6.6401	0.0255	0.0202	0.5160	
7	421.0811	158.6486	22.4192	6.6898	0.0363	0.0200	0.7254	
8	424.3243	268.9189	33.3491	6.3738	0.0271	0.0215	0.5835	
9	427.5676	139.1892	20.7136	6.4182	0.0377	0.0213	0.8040	
10	430.8108	249.4595	31.4773	6.1103	0.0287	0.0228	0.6542	
11	434.0541	119.7297	19.0324	6.1492	0.0391	0.0226	0.8855	
12	437.2973	230.0000	29.6298	5.8495	0.0302	0.0241	0.7281	
13	440.5405	340.2703	41.7826	5.8650	0.0200	0.0240	0.4805	
14	443.7838	210.5405	27.8067	5.5915	0.0318	0.0253	0.8049	
15	447.027	320.8108	39.7933	5.6151	0.0217	0.0252	0.5470	
16	450.2703	191.0811	26.0079	5.3362	0.0333	0.0266	0.8847	
17	453.5135	301.3514	37.8282	5.3680	0.0233	0.0264	0.6168	
18	456.7568	171.6216	24.2334	5.0836	0.0348	0.0278	0.9672	9
19	460.0000	281.8919	35.8875	5.1236	0.0250	0.0276	0.6897	
20	463.2432	152.1622	22.4833	4.8338	0.0362	0.0290	1.0522	7
21	466.4865	262.4324	33.9712	4.8820	0.0266	0.0288	0.7656	
22	469.7297	132.7027	20.7574	4.5868	0.0377	0.0302	1.1398	5
23	472.973	242.973	32.0791	4.6431	0.0282	0.0300	0.8442	
24	476.2162	113.2432	19.0559	4.3425	0.0391	0.0314	1.2297	3
25	479.4595	223.5135	30.2114	4.4069	0.0297	0.0311	0.9255	
26	482.7027	333.7838	42.9222	4.7865	0.0191	0.0293	0.5577	
27	485.9459	204.0541	28.3680	4.1735	0.0313	0.0323	1.0093	8
28	489.1892	314.3243	40.9125	4.5613	0.0207	0.0304	0.6299	
29	492.4324	184.5946	26.5489	3.9428	0.0328	0.0334	1.0956	6
30	495.6757	294.8649	38.9272	4.3388	0.0224	0.0315	0.7049	
31	498.9189	165.1351	24.7541	3.7149	0.0343	0.0345	1.1841	4
32	502.1622	275.4054	36.9663	4.1190	0.0241	0.0325	0.7825	
33	505.4050	145.6760	22.9837	3.4897	0.0358	0.0356	1.2748	2
34	508.6486	255.9459	35.0296	3.9020	0.0257	0.0336	0.8627	
35	511.8902	126.2160	21.2376	3.2673	0.0373	0.0367	1.3675	1
36	515.1351	236.4865	33.1173	3.6877	0.0273	0.0346	0.9452	10
37	518.3784	346.7568	46.5523	4.4234	0.0160	0.0310	0.4967	

Furthermore, let's use the hybrid of SNTO with PMOO to complete the succeeding optimal design deeply. The role of the subsequent processing of SNTO with PMOO is to contract the ranges of variables to conduct successive evaluations [**6,7**].

The uniform table $U^*_{29}(29^6)$ is used to conduct the assessment of the hybrid of SNTO with PMOO [**7**]. Table 7 gives the consequences of the succeeding evaluations.

Table 7 shows that the differences of x_1* and x_2* and δ at the 4th step and 5th step are very small, so the sequential algorithm is ended at the 5th step. In the assessment, $\delta = (P_{tp-1}-P_{tp})/P_{tp-1}$ indicates the relative variation of the maximum values of total preferable probabilities P_{tp} and P_{tp-1} in p-th step and $p-1$-th step, respectively. Thus, the final optimal consequences of this multi – objective optimization problem are y_1 = 19.9156% and y_2 = 3.0312 *mm* with δ = 2.83% at x_1* = 519.5690 °C, x_2* = *110.4310 kN*, individually. Obviously, the optimal result of present study is decisive. The results reflect the significance of the proposed method in guaranteeing the quality of product and production process comprehensively, which exhibits a good prospect in parameter design of production process in the future [**11,12**].

Table 7. Succeeding evaluation results by using $U^*_{29}(29^6)$.

	Domain	Optimum location		y_1, %	$y_{2,\ mm}$	$P_t\times10^3$	δ =
Step		x_1, °C	x_2, KN				
0	[400, 520] × [110, 350]	511.8920	126.2160	21.2376	3.2673	1.3675	
1	[460, 520] × [110, 230]	514.8280	120.3450	20.7467	3.1768	1.6808	
2	[490, 520] × [110, 170]	517.4138	115.1724	20.3159	3.0979	1.4485	13.82%
3	[510, 520] × [110, 140]	519.1380	112.5860	20.1150	3.0485	1.3086	9.66%
4	[513, 520] × [110, 125]	519.3970	111.2930	19.9953	3.0381	1.2546	4.13%
5	[515, 520] × [110, 115]	519.5690	110.4310	19.9156	3.0312	1.2191	2.83%

CONCLUSIONS

Appropriate approach of parameter design and machining of mechanical part is established by means of PMOO, some examples are dealt with, which include the designs of the grinding process with energy saving and emission reduction, and processing optimization of aluminum alloy blank hot stamping, individually. The "sequential optimization algorithm" is employed to conduct the discretization in successive optimum process for deep optimization subsequently. By performing the evaluation of preferable probability of each alternative scheme, the optimal design is thus accomplished. The optimization consequences exhibit superiority of the PMOO. The proposed method is very significant to supply a safeguard to the comprehensive quality of product and clean production, which has a bright prospect in parameter design of production optimization in the future.

DECLARATION OF COMPETING INTEREST

The authors declare no conflict of interest.

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Not applicable.

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NOTATIONS

PMOO	:	Probabilistic Multi – Objective Optimization
VIKOR	:	Vlšekriterijumska Optimizacija I KOmpromisno Resenje
TOPSIS	:	Technique of ranking Preferences by Similarity to the Ideal Solution
AHP	:	Analytical Hierarchy Process
MOORA	:	Multi-Objective Optimization on the basis of Ratio Analysis
MOO	:	multi - objective optimization
X_{ij}	:	value of utility index of the *j*th attribute in the *i*th scheme
P_{ij}	:	partial preferable probability of attribute X_{ij}
N	:	total number of candidate in scheme
M	:	number of attributes for each scheme
α_j	:	preferable probability coefficient of the *j*th attribute in beneficial type
β_j	:	preferable probability coefficient of the *j*th attribute in unbeneficial type
X_{jmin}	:	minimum value of the *j*th attribute performance utility index in unbeneficial type
X_{jmax}	:	maximum value of the *j*th attribute performance utility index in unbeneficial type
$\overline{X}_j$	:	arithmetic mean value of the *j*th performance utility index
P_i, P_t	:	overall / total preferable probability
v	:	grinding wheel linear speed
ω	:	work piece speed
s	:	feed rate
Ra	:	roughness
E	:	total energy consumption
C	:	carbon emissions
T	:	grinding time
FEM	:	finite element method
SNTO	:	sequential number-theoretic optimization
NT-nets	:	number-theoretic nets
$\delta = (P_{ti-1}-P_{ti})/P_{ti-1}$	:	relative change of total preferable probability at i-th step of SNTO