RESEARCH ARTICLE

A Probability Based Methodology for Multi Object Optimization in Material Selection

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Multi object optimization in material selection involves the satisfaction of optimizing the multi attributes simultaneously, which analogically corresponds to the simultaneous appearance of the event of the multi attributes in the viewpoint of probability theory, thus the optimization of multi – object becomes the assessment of the "joint probability" of these multi – attribute problem. Furthermore, the preferential degree of the candidate material in the material selection is reflected by the concept of preferential probability, and a quantitative approach for evaluating the partial preferential probability of each material attribute indicator and the total (joint) preferential probability of candidate material in the material selection is proposed on basis of probability theory correspondingly. In the approach, all material attribute indicators are divided into beneficial or unbeneficial types; each material attribute indicator of the candidate contributes one partial preferential probability linearly to its authorized material upon its nature of whether beneficial or unbeneficial type merely; the product of all partial preferential probabilities of a candidate makes its total preferential probability, which is the final unique index in the material selection decisively; the candidate materials can be ranked according to their total preferential probabilities, which determines the result of the selection. Furthermore, the condition of discrete input variables and the objects is extended to the case of continuous input variables and the objects. Some examples are given in detail, satisfied results are obtained.

Introduction

Material selection under condition of multi object is in fact a multi object optimization problem (MOOP), which involves the simultaneous satisfaction of multi material attributes. Actually, many problems are composed of multiple objects in daily life, which conflict and influence each other. It often encounters the optimization problem of arranging multiple tasks as best as possible at the same time in a given condition. That is, a multi-objective optimization problem. For example, when a manufacturer produces a product, it requires not only less investment, fewer workers, and lower costs, but also requires high work efficiency and high profits. This is a typical MOOP. In general, the sub-goals in the MOOP are generally contradictory, it is impossible to fully optimize each sub-goal to its own optimal value at the same time. The ultimate goal is to achieve a compromised optimization. The solution is a satisfactory solution in the overall consideration of the multiple objects.

The obvious feature of this optimization problem is that more than one optimization object is involved to be processed at the same time. MOOP has been one of the main research areas since the early 1960s, and also attracted more attentions of researchers from different backgrounds.

60 years past since the early pioneer works in MOOP, in order to obtain a systematic result many investigators have proposed a series of methods to deal with this problem [**1-5**], for example, the methods of Pareto solution, AHP (Analytical Hierarchy Process), MOORA (Multi-Objective Optimization on the basis of Ratio Analysis), VIKOR (Vlšekriterijumsko KOmpromisno Rangiranje), and TOPSIS (Technique of ranking Preferences by Similarity to the Ideal Solution), etc.

However, all above methods for MOOP adopt "additive" algorithm after parameterization and normalization of the evaluation indicators generally; some contains artificial factors [**1-5**]. From the perspective of "simultaneous optimization of multiple objects", the "additive" algorithm is equivalent to taking the form of "union" in the eyes of an analyzer of probability theory, which is the inherent shortcoming [**6**].

Contrarily, from viewpoint of probability theory, the general approach for "simultaneous optimization of multiple objects" should take the form of "intersection" of the multiple evaluation indicators [**6**]. On the other hand, since the introduction of artificial factors in some algorithms, the relevant approaches cannot be seen as a fully quantitative method in some sense. Therefore, comprehensive study in materials selection is still needed so as to develop an overall approach quantitatively.

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As to simultaneous optimization of multi- object problem, Derringer et al and Jorge et al once proposed desirability function to transfer each response variable into a desirability value [**7,8**], then all the desirability are combined by using the geometric mean method to get a single desirability value to represent the overall assessment for the combined responses. But this approach is not consistent with the essence of probability theory for simultaneous optimization of multi – objects at all.

In this paper, a probability theory methodology for MOOP in material selection is developed, which involves the introduction of a new concept of "preferential probability". The total (joint) preferential probability of a candidate material is from the overall consideration of all possible material property indicators of the candidate material. The total preferential probability of a candidate material is the unique and decisive index for the selection process finally.

Probability theory-based methodology for multi – object optimization

Basic requirement of simultaneous optimization of multi – object in probability theory

In probability theory, if an event A composes of the simultaneous appearance of some individually multiple events, for example, $A_1, A_2, A_3, \ldots, A_j, \ldots, A_m$, its probability P_t (joint probability) is the product of the partial probability of each individual event P_{Aj} [**6**], i.e.,

$$P_{At} = P_{A1} \cdot P_{A2} \cdots P_{Aj} \cdots P_{Am} = \prod_{j=1}^{m} P_{Aj} \tag{1}$$

Similarly, as to the MOOP of material selection, which involves the simultaneous satisfaction of multi material attributes, each sub-goal needs to get optimization at the same time. Lets' take each material attribute as the partial event, and the material to be selected as the total (joint) event in the material selection process, thus the MOOP of material selection can be conducted fully according to the probability theory. Now, the rest things are the evaluations of material attributes in the selection process in viewpoints of probability.

Quantitative evaluations of material attributes in the selection process in viewpoints of probability theory

1) Conception of preferential probability

Generally speaking, every material behaves various features and characteristics in different respects; each material attribute and its value represent one aspect of the material characteristics in some sense. Some material attribute indicators are beneficial to the material selection, while other attribute indicators are unbeneficial to the material selection. Actually, a practical material is an integral body of both beneficial and unbeneficial attributes with respect to material selection and utilization. Therefore, an overall consideration for material selection is necessary in the viewpoint of impersonal analysis, which results in the material selection a comprehensively systemic task.

Therefore, proper evaluation for both beneficial and unbeneficial indicators will be necessary to the material selection quantitatively.

Take the machining of a component as an example, the selections of proper cutting speed and feed quantity are the input variables, the efficiency and the life of tool are beneficial indicators for the material selection of machining, but the cost of the machining process is the unbeneficial indicator to the material selection. Therefore, both beneficial and unbeneficial indicators are all involved in this material selection process. In general, the beneficial indicators have the nature of the higher the better, and the unbeneficial indicators have the nature of the lower the better to the material selection.

In general, material attribute indicators have certain values to reflect the corresponding attributes whether beneficial or unbeneficial indicators. Therefore, as a quantitative evaluation to the term "the higher the better" and "the lower the better" for a material attribute indicator, the concept of "preferential probability" is introduced, i.e., the partial preferential degree of each material attribute indicator is directly related to its value of the material attribute indicator in the material selection process, and correspondingly the partial preferential probability of the candidate material can be used to characterize the partial preferential degree of the material in the material selection process quantitatively.

2) Quantitative evaluations of preferential probability in respect of probability theory

Because the actual value of the material attribute indicator is a quantified data of its characteristic in a sense, one could reasonably assume that the partial preferential probability of a material attribute indicator with the character of "the higher the better" (i.e., beneficial indicators) is positively correlative to its value of this material attribute indicator linearly in the viewpoint of the simplicity principle naturally, i.e.,

$$P_{ij} \propto U_{ij}. \tag{2}$$

Furthermore, Eq. (2) can be written as an equation,

$$P_{ij} = \alpha_j U_{ij},\ i = 1, 2, \ldots, n,\ j = 1, 2, \ldots, m. \tag{3}$$

In Eq.(3), U_{ij} indicates the j-*th* material attribute indicator of the i-*th* candidate material; P_{ij} represents the partial preferential probability of the beneficial material attribute indicator U_{ij}; n is the total number of candidate materials in the material group involved; m stands for the total number of material attribute indicators of each candidate material in the process; α_j indicates the normalized factor of the j-*th* material attribute indicator.

Moreover, in accordance with the general principle of probability theory [6], the partial preferential probability P_{ij} for the index i in j-th material attribute indicator can be normalized by the following summation process, i.e.,

$$\sum_{i=1}^{n} P_{ij} = \sum_{i=1}^{n} \alpha_i U_{ij} = 1 \quad (4)$$

From Eq. (4), it results in following consequence,

$$\alpha_j = 1/(n\overline{U}_j) \quad (5)$$

In Eq. (5), $\overline{U}_j$ represents for the average value of the *j-th* material attribute indicator in the material group involved.
Equivalently and comfortably, the partial preferential probability of the unbeneficial material attribute indicator U_{ij} of the candidate material is negatively correlative to its material attribute indicator linearly, i.e.,

$$P_{ij} \propto (U_{jmax} + U_{jmin} - U_{ij}),\ P_{ij} = \beta_j(U_{jmax} + U_{jmin} - U_{ij}),\quad i = 1, 2,..., n,\quad j = 1, 2,..., m \quad (6)$$

In Eq.(6), U_{jmax} and U_{jmin} express the maximum and minimum values of the material attribute indicator U_j in the material group, respectively; β_j is the normalized factor of the *j-th* material attribute indicator.
Correspondingly, by using the general principle of probability theory [**6**], it results in,

$$\beta_j = 1/[n(U_{j\max} + U_{j\min}) - n\overline{U}_j] \quad (7)$$

Furthermore, according to basic probability theory [**6**], the "simultaneous optimization of multiple objects problem" can be conducted by using Eq. (1) reasonably.

Obviously, in this evaluation the total preferential probability of a candidate material is the unique and decisive index in the material selection process comparatively.

Besides, by using above procedure and Eqs (1) through (7), the multi object optimization problem becomes a single object optimization one of the total preferential probability in the viewpoint of probability theory automatically.

Application of the probability based methodology of multi – object optimization in material selection

Material selection for a liquid nitrogen storage tank

Sarfaraz Khabbaz *et. al.*, once proposed the material selection for a liquid nitrogen storage tank [**9**], the basic requirements for the material selection include good weldability and processability, lower density and specific heat, smaller thermal expansion coefficient and thermal conductivity, adequate toughness at the operating temperature, and sufficiently strong and stiff. The properties of the candidate materials for this selection are cited in **Table 1**.

The material attribute indicators of toughness index, yield strength and young's modulus have the characteristic of the higher the better for this usage, so the these attributes belong to the beneficial indicators to the material selection; while the density, thermal expansion, thermal conductivity and specific heat are the unbeneficial material attribute indicators to this material selection, which have the characteristic of the lower the better. So, the evaluations for partial preferential probabilities of beneficial or unbeneficial material attribute indicators can be conducted according to Eqs. (3) and (5), or Eqs. (6) and (7), respectively.

Table 2 shows the results of partial preferential probability P_{ij} and the total preferential probabilities P_i evaluated for each material attribute indicators of the seven candidate materials.

Table 1. Properties of the candidate materials for the liquid nitrogen storage tank.

Material	Toughness index	Yield strength (MPa)	Young's modulus (GPa)	Density g/cm^3	Thermal expansion (10^{-6}/°C)	Thermal conductivity (J/cm^2/cm/°C/s)	Specific heat (J/g/°C)
Al 2024-T6	75.5	420	74.2	2.80	21.4	0.370	0.16
Al 5052-O	95	91	70	2.68	22.1	0.330	0.16
SS 301-FH	770	1365	189	7.90	16.9	0.040	0.08
SS 310-3AH	187	1120	210	7.90	14.4	0.030	0.08
Ti–6Al–4V	179	875	112	4.43	9.4	0.016	0.09
Inconel718	239	1190	217	8.51	11.5	0.310	0.07
70Cu–30Zn	273	200	112	8.53	19.9	0.290	0.06

Table 2. Partial and total preferential probabilities of the seven candidate materials for the liquid nitrogen storage tank

Material	Preferential probability							Total	
	Toughness index	Yield strength	Young's modulus	Density g/cm^3	Thermal expansion	Thermal conductivity	Specific heat	Pt×10^5	Rank
Al 2024-T6	0.0415	0.0798	0.0754	0.2354	0.0963	0.0121	0.0718	0.0005	6
Al 5052-O	0.0522	0.0173	0.0711	0.2388	0.0896	0.0425	0.0718	0.0004	7
SS 301-FH	0.4234	0.2595	0.1920	0.0927	0.1392	0.2629	0.1667	1.1921	1
SS 310-3AH	0.1028	0.2129	0.2134	0.0927	0.1630	0.2706	0.1667	0.3182	3
Ti–6Al–4V	0.0984	0.1663	0.1138	0.1898	0.2107	0.2812	0.1552	0.3251	2
Inconel 718	0.1314	0.2262	0.2205	0.0756	0.1907	0.0577	0.1782	0.0972	4
70Cu–30Zn	0.1501	0.0380	0.1138	0.0750	0.1106	0.0730	0.1897	0.0075	5

It can be seen from **Table 2** that the comparative result of the last column clearly shows the maximum value of total preferential probabilities *Pi* being SS 301-FH, so the optimal selection for the liquid nitrogen storage tank is the SS 301-FH, which agrees with the common knowledge [**9**].

Material selection for a flywheel

Jee *et. al.*, and Athawale *et. al.*, studied the material selection of flywheel [**10-11**]. A flywheel is a typical device that is used to restore kinetic energy in urban subway trains, automobiles, mass transit buses, wind-power generator, etc. The hazard of catastrophic failure limits its practical applications. Therefore, the main requirements in flywheel design are to restore a large amount of kinetic energy per unit mass and to resist its failure due to fatigue or brittle fracture. The evaluated attribute indicators include the specific fatigue limit of the material σ_{limit}/ρ, the specific fracture toughness (K_{IC}/ρ), price per unit mass, and fragmentability. σ_{limit} is the fatigue limit of the material and ρ is the material density; the higher σ_{limit}/ρ and K_{IC}/ρ, while lower fabrication costs and fragmentability of the flywheel material. **Table 3** shows the data of material selection for the flywheel [**10,11**].

In **Table 3**, the specific fatigue limit, specific fracture toughness, and anti - fragmentability factor are the beneficial attributes indexes, while the specific price index is an unbeneficial attribute index.

Table 3. Data of material selection for the flywheel

No.	Material	σ_{limit}/ρ $(MPa/(ton/m^3)$	K_{IC}/ρ $(MPa\cdot m^{0.5}/(ton/m^3)$	Price/ Mass ($/ton)	Anti - fragmentability
M1	300 M	100	8.61	4,200	3
M2	2024-T3	49.65	13.47	2,100	3
M3	7050-T73651	78.01	12.55	2,100	3
M4	Ti-6Al-4V	108.88	26	10,500	3
M5	E glass-epoxy FRP	70	10	2,735	9
M6	S glass-epoxy FRP	165	25	4,095	9
M7	Carbon-epoxy FRP	440.25	22.01	35,470	7
M8	Kevlar 29-epoxy FRP	242.86	28.57	11,000	7
M9	Kevlar 49-epoxy FRP	616.44	34.25	25,000	7
M10	Boron-epoxy FRP	500	23	315,000	5

Table 4 shows the results of partial preferential probability P_{ij} and the total preferential probabilities P_i evaluated for each material attribute indicators of the ten candidate materials.

Table 4. Partial and total preferential probabilities of the ten candidate materials for the flywheel.

No.	Preferential probability				Total	
	σ_{limit}/ρ	K_{IC}/ρ	Price / Mass	Anti – fragment ability	Pt×10^4	Rank
M1	0.0422	0.0423	0.1134	0.0536	0.1084	8
M2	0.0209	0.0662	0.1142	0.0536	0.0848	9
M3	0.0329	0.0617	0.1142	0.0536	0.1241	7
M4	0.0459	0.1278	0.1111	0.0536	0.3494	5
M5	0.0295	0.0492	0.1140	0.16070	0.2657	6
M6	0.0696	0.1229	0.1135	0.16070	1.5591	4
M7	0.1857	0.1082	0.1021	0.1250	2.5631	2
M8	0.1024	0.1404	0.1110	0.1250	1.9948	3
M9	0.2600	0.1683	0.1059	0.1250	5.7922	1
M10	0.2109	0.1130	0.0008	0.0893	0.0162	10

It can be seen from **Table 4** that the comparative result of the last column clearly shows the maximum value of total preferential probabilities *Pi* being M9, i.e., Kevlar 49-epoxy FRP, so the optimal selection for the flywheel is Kevlar 49-epoxy FRP, which agrees with the common knowledge [**10,11**].

Material selection for a gear

Milani et al. conducted gear material selection for high speed and high stress applications is taken from nine alternative materials [**12,13**], i.e., cast iron, ductile iron, SG iron, cast alloy steel, through hardened alloy steel, surface hardened alloy steel, carburized steel, nitride steel and through hardened carbon steel. Five material attribute indicators are measured as the overall performances of all these candidate materials, i.e., core hardness (CH), surface hardness (SH), surface fatigue limit (SFL), bending fatigue limit (BFL) and ultimate tensile strength (UTS). In these five attributes, SH, SFL, BFL and UTS are beneficial type indicators, while CH is the unbeneficial attribute indicator.

Table 5 shows the data of material selection for the gear material selection [**12,13**].

Table 5. Data of material selection for the gear material selection.

Code	Alternative	CH (Bhn)	SH (Bhn)	SFL (MPa)	BFL (MPa)	UTS (MPa)
A1	Cast iron	200	200	330	100	380
A2	Ductile iron	220	220	460	360	880
A3	SG iron	240	240	550	340	845
A4	Cast alloy steel	270	270	630	435	590
A5	Through hardened alloy steel	270	270	670	540	1190
A6	Surface hardened alloy steel	240	585	1160	680	1580
A7	Carburized steel	315	700	1500	920	2300
A8	Nitride steel	315	750	1250	760	1250
A9	Through hardened carbon steel	185	185	500	430	635

Table 6 shows the evaluation results of partial preferential probability P_{ij} and the total preferential probabilities P_i evaluated for each material attribute indicators of the nine candidate materials.

Table 6. Evaluation results of preferential probability for the candidate materials.

Code	Preferential probability					Total	
	CH	SH	SFL	BFL	UTS	Pt×10^5	Rank
A1	0.1336	0.0585	0.0468	0.0219	0.0394	0.0316	9
A2	0.1247	0.0643	0.0652	0.0789	0.0912	0.3765	7
A3	0.1158	0.0702	0.0780	0.0745	0.0876	0.4135	6
A4	0.1025	0.0789	0.0894	0.0953	0.0611	0.4211	5
A5	0.1025	0.0789	0.0950	0.1183	0.1233	1.1213	4
A6	0.1158	0.1711	0.1645	0.1490	0.1637	7.9498	2
A7	0.0824	0.2047	0.2128	0.2015	0.2383	17.2376	1
A8	0.0824	0.2193	0.1773	0.1665	0.1295	6.9098	3
A9	0.1403	0.0541	0.0709	0.0942	0.0658	0.3337	8

It can be seen from Table 6 that the comparative result of the last column clearly shows the maximum value of total preferential probabilities *Pi* being A7, i.e., Carburized steel, therefore the optimal selection for the gear is Carburized steel, it agrees with the result of Babu et al. by using grey-based fuzzy logic approach [**13**], but doesn't agree with the result of Milani *et. al.*, by employing the TOPSIS method [**12**], the difference is due to the inherent shortcoming of TOPSIS.

Application in optimization with continues function: A round log intercepting into a rectangular cross-section beam

In the last paragraphs, the input variables and the objects are discrete, while in some condition the input variables and the objects are continuous. For example, a rectangular section beam is needed to be intercepted from a log, how to choose the aspect ratio of the height and width of the section to make both strength and rigidity of the beam as greater as possible?

Solution: suppose the radius of the log is r, and the angle between the connection line from the center O to the inscribed rectangular corner A is θ, see **Fig. 1**, then the width b and height h of the rectangular section are

$$h = 3r\sin\theta, \quad b = 2r\cos\theta. \tag{8}$$

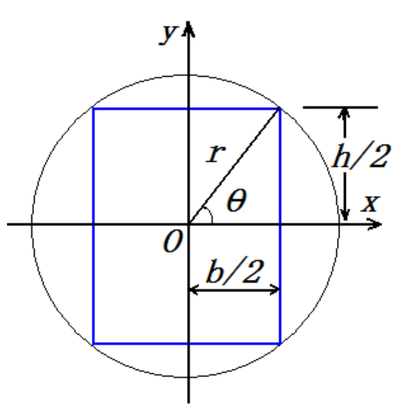

Fig. 1. Log rectangular beam.

According to the strength conditions of the beam, under the same cross-sectional area, the larger the anti-bending section coefficient W_z of the beam the better, which is with,

$$W_z = bh^2/6 = 4r^3(\cos\theta\cdot\sin^2\theta)/3 \tag{9}$$

While, according to the stiffness condition of the beam, when the cross-sectional area is the same, the larger the beam's section moment of inertia J_z the better, which is with,

$$J_z = bh^3/12 = 4r^4(\cos\theta\cdot\sin^3\theta)/3 \tag{10}$$

In this question, there involves two optimizations of both W_z and J_z with the bigger the better simultaneously.

If the anti-bending section coefficient W_z is optimized individually, it leads to an aspect ratio h / b of $2^{0.5} = 1.414$. On the other hand, if the section moment of inertia J_z is optimized individually, it leads to an aspect ratio h / b of $3^{0.5} = 1.732$. The above two individual values of optimization are clearly different.

However, in our actual condition the optimization is conducted for both anti - bending section coefficient W_z and moment of inertia J_z simultaneously, the partial preferable probabilities of both anti-bending section coefficient W_z and moment of inertia J_z can be assessed according to Eqs. (3) and (5) in principle.

Under addition of continuous functions, the "summation" in Eqs. (3) through (7) for the partial preferable probability evaluation process becomes an integral for evaluations, i.e.,

$$\int_a^b P_j(x)\cdot dx = \int_a^b \alpha_j \cdot U_j(x)\cdot dx = 1 \tag{11}$$

Eq. (11) leads to a following result,

$$\alpha_j = 1/[(b-a)\cdot\overline{U_j}], \tag{12}$$

In Eq. (12), $\overline{U_j}$ represents for the average value of the *j-th* material attribute indicator in the material group involved, $\overline{U_j} = (\int_a^b U_j(x)\cdot dx)/(b-a)$, a and b represent the domains of input variable x. Simultaneously,

$$\beta_j = 1/\{(b-a)\cdot[(U_{j\max} + U_{j\min}) - \overline{U_j}\} \tag{13}$$

As a result, the aspect ratio of h / b for the round log intercepting into a rectangular cross-section beam problem is $2.5^{0.5} = 1.581$, which is the compromised result of the individual optimizations of W_z and J_z.

Conclusion

From above results and discussion, the probability based methodology for multi – object optimization in material selection is developed, which consider all possible material attribute indicators comprehensively. All the material attribute indicators are divided into beneficial and unbeneficial types, which contribute to their partial preferential probability to the candidate material in positively or negatively correlative manners linearly. The total preferential probability of a candidate material is the product of its partial preferential probability of each material attribute indicator. The total preferential probability determines the final result of the material selection definitely and comprehensively. In addition to material selection, application prospect of this methodology in materials engineering, such as forming, casting and machining is predictive.

Conflict statement

There is no conflict of interest.

Keywords

Material selection; probability theory; linear correlation; preferential probability; multi – object optimization.

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