Coulomb Drag of Electron-Electron Interactions in GaAs Bilayer with a Non Homogeneous Dielectric Background

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Motivated by recent studies in multilayer system of dielectric background, we analytically studied the coulomb drag effect in hetero-junction of GaAs system, which consist a non-homogeneous dielectric background. By considering the weak interaction in static case, the effective dielectric function is evaluated by using Random Phase Approximation (RPA) method as RPA is a reliable study for high density regime. The effective coulomb interaction cause of electron-electron interaction in the Boltzmann regime and at low temperature limit is considered. Dependency of layer separation is effectively described by local form factor has been taken into account in effective dielectric function, the local form factor is obtained from the solution of the Poisson equation of a three-layer dielectric medium with GaAs sheets. Drag resistivity is measured numerically and analytically, where temperature and density dependence are investigated and compared to 2DEG-2DEG double-layer structures theoretically and experimentally at very low temperature.

Introduction

In such typical experiments, Coulomb drag (CD) is a transport effect in bilayer system, current is applied by a layer which is known as active layer and an induced voltage (drag voltage) is found across the other layer known as passive or drag layer. Where both the layers are electrically isolated. With the boundary condition, no current flow in passive layer but an induced voltage is found as shown in **Fig. (1a)**. The induced voltage V_D caused by e-e interaction between the coupled layers rises to a frictional force that drags the passive layer electrons. The ratio of the induced voltage to the driving current which give the measurement of e-e interaction between the coupled layers, termed as the drag resistivity ρ_D. The resulting non- local ρ_D is a direct problem of the rate of momentum transfer rate between the two layers by interaction of electron-electron, electron-hole, electron impurity, etc.

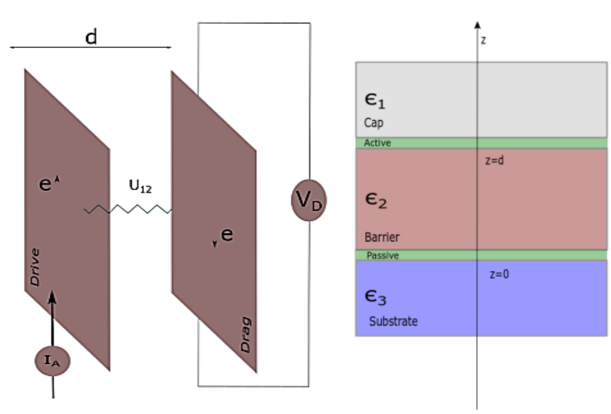
$$\rho_D = \frac{E_{drag}}{j_{active}} \qquad (1)$$

Ultimately, Drag effect in bilayer electron systems is induced by fluctuations of the individual carrier density of the two layers [**1**]. However, a electric field along the normal direction is generated in a finite plane with uniformly distributed electric charge that doesn't exert any lateral force on the carriers in drag layer. If both the layers are in the condition of Fermi liquid state, then the phase space argument [**2**] yields dependence of temperatures on drag rate as $\tau_D^{-1} \propto T^2$. Detailed analysis contain from other experimental data yields the existence of additional mechanisms leading to frictional drag, such as phonons-mediated indirect interlayer interaction [**3-6**], Plasmon effects [**7,8**], and thermo electric phenomena [**9**].

Consequently, by the time of experiment on coulomb drag effect was devoted to the numerically and quantitative measurement, for measuring the strength of the interactions due to induced field between quasi particle sub-systems in different-different semiconductor devices as GaAs quantum wells with the p-modulation doped structure [**10,11**], electron-electron bilayer system of 2D-3D AlGaAs/GaAs hetero-structures [**9,12**], 2D electron-electron in AlGaAs/GaAs DQW [**2,9,13-15**], respectively. Transport properties of two dimensional (2D) electron and hole systems have amassed a great interest. Theoretically, it may be realized that mutual Coulomb scattering between the coupled layers due to the results of the exchange of momentum $\hbar q$ and energy $\hbar\omega$ [**16,17**]. Following the ground breaking experimental work in the AlGaAs/GaAs double quantum wells (DQW) [**2,15**], there are so many articles of coulomb drag effect. Whereas the Coulomb mechanism [**1,18-20**] captures the effect's most qualitative features, such as momentum transfer $\hbar q$ may also contributed. Some of suggested the scattering phenemeno involved acoustic [**21**] and optical [**22**] phonons, Plasmon effects, and coupled Plasmon phonon modes [**23**]. Drag effect became an important part for measuring the many body properties, in the index of standard tool box in condensed matter physics of matter, after the turn of the century. It had been used to analyze the properties of e-e interactions in low density regime of 2D electron systems [**24**], metal-insulator-transition signatures in diluted 2D hole [**25-28**], quantum coherence of electrons [**29-31**] and composite fermions [**32**], excitons effects in e-h bilayers [**33-35**]. Interlayer interaction and associated transport properties were measured in hybrid devices consisting quantum wire QW and quantum dot QD [**36**], a SC film and a 2D electron gas [**37**], Si metal oxide semiconductor systems [**38**], quantum point contacts [**39**], insulating SiNb films [**40**], e-h interaction in QW [**41**], grapheme mono-layers [**31,42,43**], and hybrid graphene semiconductor systems [**44**].

On the theory side, there is an even richer field of CD which yields the variety of suggested, extensions and generalizations in the field the main coulomb drag problem. The Coulomb drag theory was extended to multilayer between two 2DEGs, which is an intriguing electronic system which typically consists of three or more layers based on Graphene and GaAs 2DEGs [**45, 46-49,61**]. Though there are just a few experimental works on CD [**31,50**]. For the simplest such structure, the double quantum well (DQW), Coulomb drag in bilayer systems is a very interesting phenomenon. We consider two intrinsic DQW of GaAs separated by a barrier SiO_2 and Al_2O_3. The theory [**51**] deals with the case of a large interlayer separation $k_F d \gg 1$, where d is the thickness of the spacer and k_F is the material's Fermi wave vector. It is a well established result for the low temperature, large interlayer separation and high density limit that the ρ_D between two 2DEGs is proportional to temperature as T^2 and with the interlayer separation as d^{-4}.

Our evolution doesn't depend on the relation of energy-dispersion, structure of wave function and momentum. The article is organized as follows: current section is Introduction, after this model and theoretical formalism, Results and discussion are presented, and the paper is concluded with a brief summary.



(a) (b)

Fig. 1. Schematic drawing of the experimental geometry. The current I_A is applied from the active layer, and the voltage difference V_D is induced in passive layer as shown in Fig. (1a). Schematic geometry illustration considered in Fig. (1b). The 2DEG is located within a QW and is defined by active and passive for its position along the z direction. The structure's relative dielectric constants are given by $\epsilon_1, \epsilon_2, \epsilon_3$ as shown in Fig. (1b).

Theoretical formalism

With consideration the tunneling does not occur, the interlayer effective interaction $(U_{12}(q))$ is caused by electron-electron static interaction in steady state and homogeneous system. The drag resistivity ρ_D evaluated by the using the Boltzmann's kinetic equation [**18,19,52, 53**], the memory function formalism [**1**], and Kubo's formulism [**20,53,54**]. The electron transport based on kinetic theory approach has certain advantages over other approaches as it allows one to account on equal footing for inter-layer and intra-layer interactions, and in principle, it may be extended to conditions of non-equilibrium. For estimating of the drag response, the non-equilibrium part of the electron distribution function is important, but there are also corresponding corrections to the electron polarization function that leads to drag resistivity contributions. In contrast, the functional form of the polarization function is strongly impacted by intralayer collisions [**55**]. In this report we calculate the ρ_D for a system of dielectric environment, composed as air/passive/barrier/active/substrate as shown in **Fig. (1b)**.

The system may be interpret as the interaction is considered weak for large distance, high density, and low temperature limit, where tunneling, exchange, and any correlation effect haven't taken.

Drag Resistivity ρ_D

We begin the study of drag resistivity with a general expression in interlayer Coulomb interactions [**2,18,19, 24,47,49**], (where both layers are identical and symmetric),

$$\rho_D = -\frac{\hbar^2}{8\pi^2 e^2 n_1 n_2 k_B T}\int_0^{\infty} dq\, q^3 |U_{12}(q)|^2 \times \int_0^{\infty} d\omega \frac{|\Im\chi_1(q,\omega)||\Im\chi_2(q,\omega)|}{sinh^2(\frac{\hbar\omega}{2k_B T})} \tag{2}$$

To evaluate the drag resistivity, we have a general solved equation, nonlinear susceptibility function and effective interlayer interaction are main function. This gives the dependency of temperature T, density n, interlayer distance d, etc.

Nonlinear susceptibility function

With following the general equation of the nonlinear susceptibility function $\chi(q,\omega)$ in low frequency regime (Ballistic regime) $\omega\tau \gg 1$ or $k_F \ell \gg 1$. The Nonlinear susceptibility function of the individual layer response functions $\chi_{1(2)}(q,\omega)$ [**18,47,56**] may be written as,

$$\chi_{1(2)}(q,\omega) = -\int \frac{dk_{1(2)}}{4\pi^2}\frac{(f^0(\varepsilon_{1(2)}) - f^0(\varepsilon_{1(2)} \pm \hbar\omega))}{\varepsilon_{1(2)} - \varepsilon_{1(2)'} \pm \hbar\omega - i\delta} \tag{3a}$$

$$\chi_{1(2)}(q,\omega) = -\nu[1 - \frac{\theta(q/k_F - 2)\sqrt{(q/k_F)^2 - 4}}{q/k_F} \tag{3b}$$

Second equation of Eq. (3b) is the response of polarisation function for static case, $\omega \rightarrow 0$. In this case, the imaginary part of the non interacting and nonlinear susceptibility function as [**19,20,57**],

$$\Im\chi(q,\omega) = -\int \frac{dk_1}{4\pi^2}\left(f^0(\varepsilon_1) - f^0(\varepsilon_1 + \hbar\omega)\right) \times \delta(\varepsilon_1 + \varepsilon_{1'} - \hbar\omega) \quad (4a)$$

$$\Im\chi(q,\omega) = \frac{2mv\omega}{\hbar q}\frac{\theta(2k_F - q)}{\sqrt{(2k_F)^2 - q^2}} \quad (4b)$$

$\theta(2k_F - q)$ is Heaviside step function, ν is density of state. The temperature dependence of ρ_D, in this regime, is entirely determined by the denominator $sinh^2(\frac{\hbar\omega}{2k_BT})$ in Eq. (2), which restricts the integral in frequencies, $\hbar\omega < 2k_BT$.

Effective interaction [$U_{12}(q)$]

For measuring the screening properties of the conduction electrons in the layers, we employ the standard tool box of solving the Dyson equation for the coupled layer system within the random phase approximation (RPA) [**20, 58**]. This finally presents the standard equation of interlayer interaction as,

$$U_{12}(q) = \frac{U_{12}^0(q)}{\varepsilon_{RPA}(q)} \quad (5)$$

The effective interaction is obtained by evaluating the Eq. (5) for the interacting field due to a point source situated in one of the two layers. Dielectric tensor $\varepsilon_{RPA}(q)$,

$$\varepsilon_{RPA}(q) = \left(1 + U_{11}^0(q)\chi_1(q,\omega)\right)\left(1 + U_{22}^0(q)\chi_2(q,\omega)\right) - (U_{12}^0(q))^2\chi_1(q,\omega)\chi_2(q,\omega) \quad (6)$$

Local form factors

$U_{ii}^0(q)$ and $U_{ij}^0(q)$ are called bare intra and interlayer interaction respectively, and local form factor (LFF) $F_{ij}(qd)$ are key equations. To evaluating the form factor $F_{ij}(qd)$, electrostatic problem needs to solve with considering the different screening by the substrate, barrier, and air [**46,47,59**]. Let the dielectric environment where the dielectric constants of the layers are,

$$\epsilon = \begin{cases} \epsilon_1 & z > d \\ \epsilon_2 & 0 < z < d \\ \epsilon_3 & z < 0 \end{cases} \quad (7)$$

Using the solution of Poisson equation, the $U_{11}(q)$, $U_{22}(q)$ and $U_{12}(q)$ by [**18,46,47,58,**],

$$U_{ij}(q) = \frac{8\pi e^2}{q}F_{ij}(q) \quad (8)$$

The bare inter- and intralayer potentials may be evaluated by the Poisson equation. With introduction of the Fourier transform of coulomb potential $\varphi(\boldsymbol{q}; z, z')$ along the z direction. The form factors for non finite width [**46,47,59**],

$$F_{11}(d) = \frac{\epsilon_2 \exp(qd)[\epsilon_2\cosh(qd) + \epsilon_3\sinh(qd)]}{[(\epsilon_1+\epsilon_2)(\epsilon_3+\epsilon_2)\exp(2qd) - (\epsilon_1-\epsilon_2)(\epsilon_3-\epsilon_2)]} \quad (9a)$$

$$F_{22}(d) = \frac{\epsilon_2 \exp(qd)[\epsilon_2\cosh(qd) + \epsilon_1\sinh(qd)]}{[(\epsilon_1+\epsilon_2)(\epsilon_3+\epsilon_2)\exp(2qd) - (\epsilon_1-\epsilon_2)(\epsilon_3-\epsilon_2)]} \quad (9b)$$

$$F_{12}(d) = \frac{\epsilon_2 \exp(qd)}{[(\epsilon_1+\epsilon_2)(\epsilon_3+\epsilon_2)\exp(2qd) - (\epsilon_1-\epsilon_2)(\epsilon_3-\epsilon_2)]} \quad (9c)$$

Result and Discussion

We have used a general computational scheme and model to describe non-local transport in interactively coupled double layer systems at low temperature and ballistic regime We don't have any experimental and theoretical data corresponding to our system and compare our results theoretically [**2,18,19,24,47,49,61**] and experimentally with [**29**] of similar results based on non-homogeneous dielectric environment where. Present approach is based on the Boltzmann kinetic theory, the route to existing formulation to attain the Eq. (2). With the low temperature limit $T \ll T_F$, we have $\chi_{1(2)} \propto \omega$, the integration in Eq. (2) reads $\int_0^\infty d\omega \frac{\omega^2}{sinh^2(\frac{\hbar\omega}{2k_BT})} \propto T^3$, which gives the T^2 dependency of the ρ_D in the limit of $T \ll T_F$. Note that this system is taken under the large interlayer distance $k_Fd \gg 1$. In the limit $k_Fd \gg 1$ drag resistivity behave with concentration and others as [**2,18, 19,24,47,49,61**],

$$\rho_D \propto \frac{T^2}{n^3d^4} \quad (10)$$

This is the same temperature dependence found for the transresistivity in ballistic 2DEG-2DEG bilayer [**1,2, 18-20**]. The nature of T^2 is not dependent of the relation of the energy dispersion, transport time and wave function overlap factors. However, the behavior might be changed if one provides corrections to the ρ_D due to finite and non-finite temperature correction of susceptibility function, and in higher order terms in the interlayer interaction [**60**].

In this work, we assumed two 2D-GaAs quantum wells separated by SiO_2 and Al_2O_3 as a barrier, AlGaAs use as substrate. Here we consider both the layers are identical, symmetry, and homogeneous, such as carrier concentration $n_1 = n_2 = n$ and effective mass $m_1 = m_2 = m$ are same. In our calculation we use material parameter appropriate for GaAs systems such as, effective mass m=0.067m_e. Where $m_e = 9.1 \times 10^{-31}\,Kg$. mass of electron, dielectric constant of substrate, barrier and air are 13, 4 and 1, respectively. A theoretical calculation have presented to measure the resistivity based on the RPA method at electron density $n = 2 \times 10^{11}\,cm^{-2}$ as RPA method is reliable method for high density regime $k_Fd \gg 1$ [**2,24,46,47,61**], as shown in **Fig. 2, Fig. 3**. The carrier concentration n dependency with ρ_D at T=10 K is shown as in Fig. (3). In this work, the inter-layer separation is set at d=20 nm, 30 nm, the density is $n = 2 \times 10^{11}\,cm^{-2}$, which tends to almost similar behavior as $\rho_D \propto n^{-3}$ and T^2 also shown in ref [**2,24,29**], we didn't consider the giants fluctuations. We have measured closed data with [**2,24, 29,61**] even our results are in good agreement with others [**2,18,19,24,47,49,61**]. Such as for the parameters, T=5 k, d=28 nm, $\epsilon = 13$, and $n \sim 3.1 \times 10^{10}\ cm^{-2}$, $\rho_D \sim 16.78\ \Omega/\mathrm{cm}^2$ compare to [**62**] without considering the different dielectric distribution function Eq. (7), as we have conceded better results compare to [**2,29,61**] results as shown in **Fig. 2**, with considering the different dielectric distribution function, T = 5 k, d = 30 nm, $n \sim 2 \times 10^{11}\ cm^{-2}$, dielectric constants used as Eq. (7), and $\rho_D \sim 51\ m\Omega$.

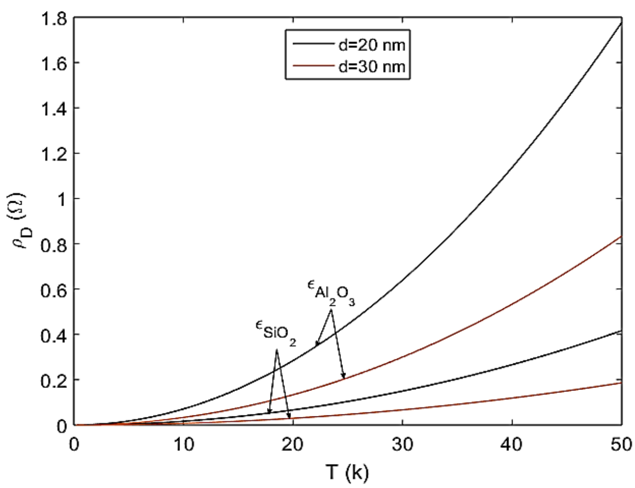


Fig. 2. The schematic diagram show the dependence of the resistivity ρ_D with the temperature T (at low temperature ($T \ll T_F$) as shown in figure for the GaAs quantum well with carrier concentration $n = 2 \times 10^{11}\ cm^{-2}$. The curves also show the dependence of the dielectric constant of the barrier (SiO_2 ($\epsilon_2 = 4$) and Al_2O_3 ($\epsilon_2 = 4$)) and interlayer distance (d=20 nm and 30 nm).

In this section, the drag resistivity ρ_D behaviour as a function of charge carrier concentration n for $k_F d \gg 1$ is shown as in **Fig. 3**, and as a function of temperature is shown as in **Fig. 2**.

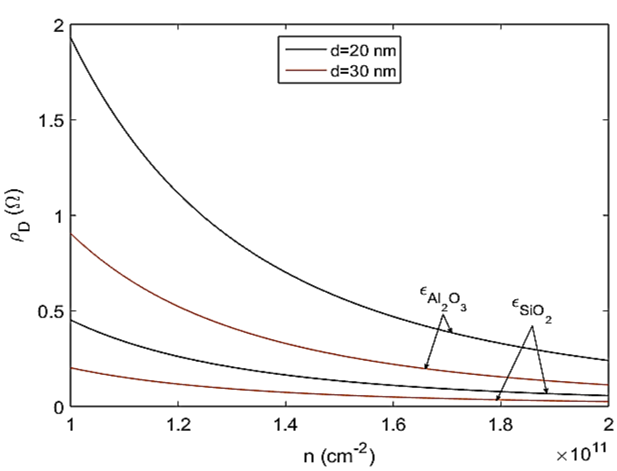


Fig. 3. The schematic diagram show the dependence of the resistivity ρ_D with the concentration for the GaAs quantum well at temperature T=10 k.

The second part of Eq. (2) as q integral, effective interaction plays a crucial role for ρ_D measurement with the values of d and dielectric constants of the layers. Local form factor are the function of layer separation and dielectric constant etc., and also shows the layer separation dependency [**61**] as shown in **Fig. 4**.

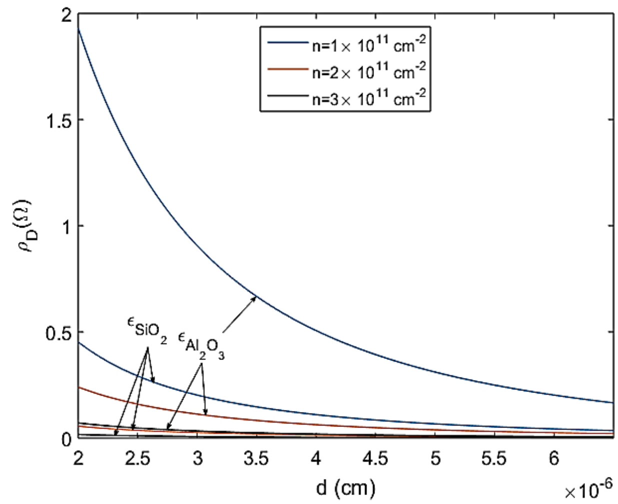


Fig. 4. Schematic diagram show the behaviour of the ρ_D with interlayer separation d for the system of $air/GaAs/SiO_2/GaAs/AlGaAs$ as shown.

The resistivity ρ_D at the low temperature T, large separation d, and high density n limit should behave as d^{-4} by the general arguments here. Moreover, a dependence of d^{-6} was obtained in [**52**] and within the same limit. In [**53**], the d^{-6} results are due to estimating the function $\vec{v}_{\vec{k},\lambda}, \tau_{\vec{k},\lambda}$ used in [**52**] with q^2, as $\vec{v}_{\vec{k},\lambda} = \hbar q/m$ and $\tau_{\vec{k},\lambda}$ is linearly dependent of q. While it should be independent of q.

Conclusion

In this section, we discussed the work done in this article, with a short note. Here, we measure the CD numerically and analitically at low temperature in a bilayer system consisting of 2DEG layer, may be regarded as being in the Boltzmann regime. The investigations for a dielectric environment make the study of coulomb drag phenomena special. As the results are in good agreement compare to the system of a simple coupled layer. We wrote down the formulas to explain such a system's the transresistivity at low temperatures and high density dependence using RPA method [**46,47**]. We have shown that the drag resistivity should always act as T^2 in low-temperature $T_F \gg T$, d^{-4} in large inter layer separation $qd \gg 1$ and high density limit, $k_F d \gg 1$ [**61**]. Hence, it is more common outcome as others. The general expression evaluated for the non-interacting susceptibility function is central to this fact, in the low temperature and low frequency regime which covers the temperature dependency of resistivity in transport phenomena.

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Author's contributions

Conceived the plan: SKU, LKS; Data analysis: SKU, LKS; Wrote the paper: SKU. Authors have no competing financial interests.

Keywords

Fermi-liquid regime, non-homogeneous system, dielectric background, RPA method.

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