Predicting Tensile Behaviour of Bulk Bamboo using Weibull Statistics for Progressive Failure

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Bamboo is a natural composite material consisting of unidirectional fibre bundles, oriented along axial direction, embedded in soft parenchymatous matrix. The bundles are arranged such that the fibre density (or fibre volume fraction) varies from outer to inner periphery of bamboo shoot. The gradation in volume fraction of unidirectional fibre bundles qualifies bamboo as a typical radially graded transversely isotropic material. Being largely a cellulosic material, the fibre bundles have high tensile strength. However, there is great dispersion of these properties. In this work, an attempt is made to model the progressive failure of fibre bundles to predict the failure strength of bulk bamboo in uniaxial tension. A two-parameter Weibull distribution is proposed to analyse the strengths of fibre bundles having different cross-section areas. Tension tests are performed on fibre bundles, selected from different fibre density regions in the transverse cross-section of bamboo, for determining statistical parameters. The results highlight the close resemblance between the Weibull probability distribution of the experimental results on fibre bundles and overall mechanical behaviour of the bulk bamboo. Thus, the use of Weibull parameters is established for predicting the strength of bulk bamboo from fibre bundle testing of different cross-section areas.

Introduction

Bamboo is a monocotyledonous plant and a true grass *Poaceae* which is widely used in various traditional and industrial applications (http://nbm.nic.in/Achievement/Handbook%20on%20Bamboo.pdf) as well as utilized to design robust structures [**1**]. It resembles a typical fibre reinforced composite where the fibre bundles are reinforced in parenchymatous matrix tissue (see, **Fig. 1**). Moreover, in bulk bamboo, the density or volume fraction of the fibre bundles continuously varies from 22% at inner periphery to 62% at the outer. It has been shown that the variation of the longitudinal stiffness closely correlates with the variation in the fibre bundle density [**2**]. The microfibrils are arranged in spiral fashion in the matrix. The average orientation of the microfibrils about the longitudinal axis which is known as mean microfibril angle (MFA) is one of the key parameters governing the longitudinal stiffness of bulk bamboo. The particular arrangement of fibre bundles on cross-section (see, **Fig. 1(b)**) makes bulk bamboo a transversely isotropic material.

It has been found that the longitudinal Young's modulus and strength of bamboo are linearly increased from the inner to outer side [**3**]. Bamboo, in particular, the species *Dendrocalamus strictus*, has an average Young's modulus of 10 GPa [**2**]. This qualifies bamboo a superior structural material. The Young's modulus for other common species is of the same order. For example, the mechanical stiffness and strength of bulk bamboo, for species *Phyllostachys edulis* and *Phyllostachys pubescens*, have been exhaustively worked upon in the former [**4**] and in the latter [**5**]. Mechanical properties of bamboo internodes are transversely isotropic due to axial alignment of fibre bundles. However, in nodes, the fibre bundles are randomly distributed, thereby producing isotropic properties.

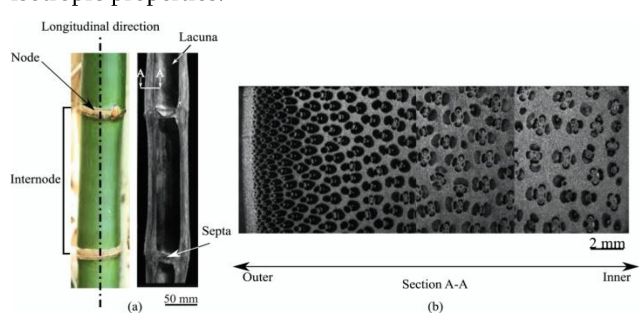

Fig. 1. (a) Longitudinal section of bamboo. (b) Section A-A showing graded distribution of fibre bundles, denser at the outer radial location of bamboo.

Fibre bundle is the basic stiffening element of bamboo [**6**]; which indicates that most of the mechanical properties of bamboo are directly derived from the fibres' properties and fibre strength distribution, as well [**7**]. It has been observed that typical bamboo fibres are brittle [**8**] and to study the brittle fracture of fibres, conventional theory has been used. Recently, a statistical weakest link theory is introduced which assumes that a material is made of small elements which are linked together and it can be divided into its elements. Further, it is assumed that the fracture initiates at the weakest link and failure of the material occurs [**9**].

Bamboo fibre bundles are not uniform in size and shape (see, **Fig. 1(b)**). Moreover, fibres within the bundles are of different sizes, length varying between 1 mm to 5 mm (average 2.8 mm) and diameter 14 µm to 27 µm (average 20 µm) [**6**]. Due to the randomness of this extent, the strength of fibre bundle and bulk bamboo in turn, is no more uniform. Hence, in addition to fibre length and diameter, the strength of bulk bamboo depends on the distribution of defects within the fibre.

There have been many attempts trying to predict the fibre strength. The optimisation of New Zealand grown hemp fibre has been investigated for inclusion in composite using Weibull distribution function obtained for single fibre strength [**10**]. In another work, the causes of discrepancies in statistical strength distribution of commercial E-glass fibres have been studied [**11**]. In multiple data set (MDS) weak-link scaling, jute fibres in tension have been analysed and predicted better correlation with the experimental data [**12**].

Weibull probability distribution has been commonly used in hazard, survival or reliability studies [**13**]. It has been shown that the tensile strength of natural fibre bundle is very well predicted in Weibull probability distribution. This probability distribution has been shown to be improved by including the normalised length or volume of each fibre [**14**]. In a separate work, both strength and elasticity modulus are described using the same Weibull probability distribution [**15**].

As far as bamboo is concerned, it is clear from the results of above studies that the Weibull probability statistics can be also be used to represent the properties of the single bamboo fibre bundles. A progressive failure model for bamboo fibre bundles has been implemented to determine the minimum size of cross-section of fibre bundle for mapping an average mechanical response [**16**]. Although, all of these studies clearly justify the implementation of Weibull statistics and predict behaviour of single fibre bundle, the overall behaviour of bulk bamboo is not well established.

It is intuitive, within the framework of Weibull statistics, to predict the failure strength of bulk bamboo in uniaxial tension. In this work, for determining statistical parameters, tension tests are performed on fibre bundles having different cross-section areas, selected from different fibre density regions in the transverse cross-section of bamboo. The statistical results are then compared with the experimental results of uniaxial tension of bulk bamboo.

The paper is organised in the following manner. The experimental procedure is discussed in Sec. 2. Statistical distribution on the properties of bamboo fibre bundles is derived making use of Weibull statistics [**17**] in Sec. 3. In Sec. 4, statistical modelling of mechanical response of the fibre bundle is deduced. The representative tensile behaviour of bulk bamboo is discussed in Sec. 5. The results are discussed in Sec. 6. The paper is concluded in Sec. 7 with findings of the Weibull statistics.

Experimental details

For the experiments on bamboo fibre bundles reported in this study, a bamboo culm of local variety was obtained from the botanical nursery of IIT Kanpur. It was kept under roof for natural seasoning to ensure it to be free from moisture. One internode was selected for making tensile specimens. For experiments, two kinds of specimens were prepared from this internode.

The first set of specimens as shown in **Fig. 2(a)** was prepared for uniaxial tensile test on fibre bundles. Following the procedure of chemical extraction for fibre bundles of bamboo (reported elsewhere, [**2**]), 75 fibre bundles were selected for the tests[1]. A bundle contains hundreds of fibres whose shape of the cross-section depends on the location of fibre bundle in the cross-section. The bundles further were naturally dried and then cut into pieces 26±1 mm in length. For testing, the ends of bundles were bonded with epoxy resin with the help of hard paper. A micro-tensile stage (Deben MICROTEST, UK) with a 300 N load cell was used for tensile tests of fibre bundles.

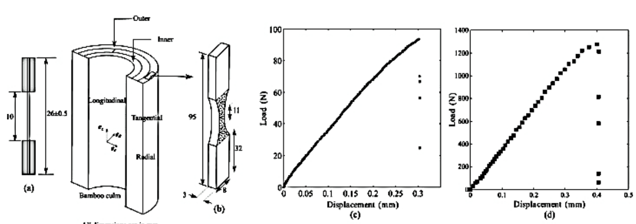

Fig. 2. A bamboo culm showing the test specimens prepared from it. (a) Fibre bundle tensile specimen. (b) Tensile specimen obtained from strips extracted from transverse cross-section. Note, the dots represent speckle pattern used for DIC. Typical force-displacement response obtained from tensile tests on fibre bundle and bamboo strips are shown in (c) and (d) respectively.

The second set of specimens (shown in **Fig. 2(b)**) was prepared for uniaxial tensile test of bulk bamboo. The specimens were prepared following ASTM D143 recommendations [**18**] with the length in the longitudinal direction and taken out from inner and outer periphery over the cross-section. A universal testing machine fitted with a 10 kN load cell was used to conduct tension tests on bulk bamboo specimens. Digital image correlation (DIC) technique along with a commercial software Vic-2D (Correlated Solutions, USA) for accurate measurement of displacements on the bulk specimen surface was used. The experiments mentioned in this work are carried out at the High Speed Experimental Mechanics Laboratory at IIT Kanpur. In all cases, the cross-head speeds were maintained at 1 mm/min. Also, to avoid the errors in analysis, specimens failed at the clamps were excluded

[1] The procedure laid down in **[2]** does not produce a single fibre. Hence, the tests were carried out on fibre bundles.

because such failure does not represent a successful test. For the tensile test to be acceptable, the specimen should break within gauge length.

A typical force-displacement response obtained from both the tests is shown in **Fig. 2(c)** and **Fig. 2(d)**. It is apparent from the plots that tensile force monotonously increased with strain until the peak force is achieved. At this point, the material fails without further yielding. The fibre bundle shows typical linear elastic brittle behaviour. Later in Sec. 5, we will discuss how tensile behaviour of bulk bamboo is predicted from independent fibre bundles. Results of the tensile tests on fibre bundles thus determined are shown in **Table 1**, where A_i, ε_i^u and σ_i^u is cross-section area, ultimate strain and strength of individual fibre bundle i.

Table 1. Statistical representation of the results of the tensile tests on fibre bundles of bamboo.

Variable (x)	Dimension	Minimum (min(x))	Maximum (max(x))	Mean ($\bar{x}$)	St. dev. ($\sigma(x)$)	Skewness ($\gamma_1(x)$)
A_i	(mm^2)	0.1238	0.2703	0.1896	0.0433	0.1147
ε_i^u	(m/m)	0.0267	0.0422	0.035	0.051	-0.1477
σ_i^u	(N/mm^2)	182.7612	805.1227	508.3507	217.1459	-0.2088

Weibull statistics

It is clear from **Fig. 2(c)** that bamboo fibres typically exhibit brittle failure and have variation in their strength which is governed by microstructural defects acting as stress raisers. Also, these defects are dispersed or distributed randomly along the fibre length. Thus, the properties are not able to be described through a deterministic model. The progressive failure of composite sandwich beam has been analysed where the authors have implemented a material model using maximum stress failure criterion [**19**]. In the present work, this model is not suitable as it requires knowledge of all stiffness constants. Hence, a probabilistic model like Weibull statistics is more suited to these kinds of representations [**20**]. Weibull statistics has been shown to be the best tool for characterization of fibre strength having scatter and random variation [**17, 21**].

The classical two-parameter Weibull cumulative density function is described by

$$P(x) = 1 - \exp\left[-\left(\frac{x}{\beta}\right)^{\alpha}\right] \qquad (1)$$

where x is the variable to be described (ε_i^u or A_i) and α and β are shape and scale parameters respectively. However, as the fibre bundle's strength depends on respective cross-sectional area, in the present study, a modified Weibull probability distribution [**22**] is used to describe fibre bundle strength σ_i^u

$$P(\sigma_i^u, A_i) = 1 - \exp\left[-\left(\frac{A_i}{A_0}\frac{\sigma_i^u}{\beta}\right)^{\alpha}\right] \qquad (2)$$

where A_0 is the average cross section area of the bundle, taken as the reference value.

The shape and scale parameters (α and β) can be obtained by linearising Eq. 1 as

$$\ln[-\ln(1 - P(x)] = \alpha \ln(x) - \alpha \ln(\beta). \qquad (3)$$

The value of the probability $P(x)$ is estimated such that the parameters α and β fit experimental data. Commonly, these parameters are estimated using probability index functions [**23**].

In the present study, the probability function used is:

$$P(x) = \frac{j - 0.5}{n} \qquad (4)$$

where j is the rank of the jth data point and n is the number of data points. Later, the values of Weibull Parameters α and β are obtained from a linear regression of Y = AX + B. The algorithm for determining Weibull Parameters α and β is reproduced in Appendix A.

The exercise mentioned above yields a good fit of the probability distribution to cross-section area A_i, ultimate strain ε_i^u and strength σ_i^u obtained from experiments on fibre bundles. The results of regression analysis are plotted in **Fig. 3**.

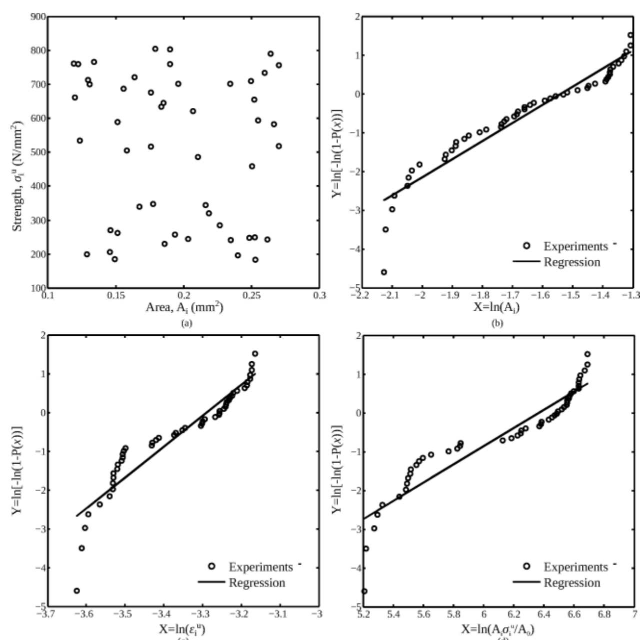

Fig. 3. (a) Statistical presentation of fiber bundle's strength expressed as a function of cross-section area of the bundle. Weibull fitting curves for cross-section area, ultimate strain and strength are shown in (b), (c) and (d) respectively.

Statistical modelling of the tensile behaviour of bulk bamboo

As mentioned in Sec. 1, bulk bamboo is composed of fibre bundles embedded in parenchymatous matrix. Each fibre bundle in turn is made of closely packed single fibres. The tensile behaviour of a fibre bundle (as shown in Fig. 2(c)) depicts a typical brittle failure which can be represented by a relationship between strength (σ_i^u) and ultimate strain (ε_i^u). This relationship can be approximated as linear until the tensile strength is achieved. Afterwards, the load drops down while strain increases as shown in **Fig. 4(a)**). Since, the tensile behaviour of bamboo predominantly depends on the strength of bamboo [**2**], the contribution of matrix is safely neglected. Also, the slip between the fibre bundles and matrix is not modelled here. The prediction of tensile strength of bulk bamboo can be obtained using a progressive failure model of fibre bundles in the bulk specimen. We now develop a model based on the mixing theory [**24**] and Weibull probability distribution worked out in previous section. It is considered that the strain of each fibre bundle $\varepsilon^{\text{bulk}}$ is the same and the strength of bulk bamboo σ^{bulk} is equal to the sum of the fibre bundle strengths weighted with the area fraction k_i, that is,

$$\varepsilon^{\text{bulk}} = \varepsilon_1 = \varepsilon_2 = \cdots = \varepsilon_{\text{i}} = \cdots \varepsilon_{\text{n}}. \quad (5)$$

$$\sigma^{\text{bulk}} = \text{k}_1\sigma_1 = \text{k}_2\sigma_2 = \cdots = \text{k}_{\text{i}}\sigma_{\text{i}} = \cdots \text{k}_{\text{n}}\sigma_{\text{n}}. \quad (6)$$

The shape and the scale parameters of Weibull probability distributions are obtained from the experimental tensile test of the fibre bundles, as it is indicated in Sec. 3 and are summarized in **Table 2**. Each numerical realisation generates a random set of ultimate strains $\varepsilon_{\text{i}}^{\text{u}}$ and strengths $\sigma_{\text{i}}^{\text{u}}$ at the failure for each fibre bundle i. Combining the progressive failure of all fibre bundles, a non-linear response is generated as shown in in **Fig. 4(b)**. The procedure followed in this progressive model is reproduced in Appendix B. At (i+1)$^{\text{th}}$ loading step in the failure of bulk specimen, the fibre bundles 1 to i are broken, and the fibre bundles (i+1) to n are still elastic.

Table 2. Statistical representation of the results of the tensile tests on fibre bundles of bamboo.

Variable (x)	Dimension	A	B	α	β
A_i	(mm^2)	5.1438	8.1171	5.1438	0.2064
ε_i^u	(m/m)	10.5799	34.7595	10.5799	0.0374
σ_i^u	(N/mm^2)	2.78	-17.3390	2.78	511.3906

It is noteworthy that there is always a linear relationship between the strength and the failure strain but there is a difference in mechanical properties which are associated with the corresponding Weibull probability distribution. To explain it further, a response for 50 fibre bundles with Weibull properties derived in this work is shown in **Fig. 4(c)**. The plot indicates the effect of the Weibull distribution having a wide dispersion in the response.

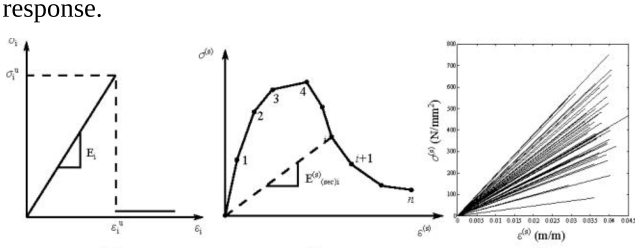

Fig. 4. Stress-strain relationship for (a) fibre bundle and (b) bulk bamboo according to the progressive failure model. Mechanical response for 50 fibre bundles with Weibull properties in **Table 2** is shown in (c).

Representative tensile behaviour of bulk bamboo

The present study is aimed at predicting the tensile behaviour of bulk bamboo which consists of stiff fibre bundles embedded in soft parenchymatous matrix. For representation of tensile behaviour, the specimens used for uni-axial tensile tests as shown in Fig. 2(a) were selected. The fibre volume fraction of these specimens was calculated to obtain fibre bundle density as 2.0 and 3.8 bundles/mm^2 giving 18 and 34 fibre bundles respectively in inner and outer bulk specimens. With the numbers of fibre bundles at hand and applying the procedure laid down in Appendix B for 10 random realisations, the representative tensile behaviour of bulk bamboo is predicted. The responses thus obtained are plotted in **Fig. 5**.

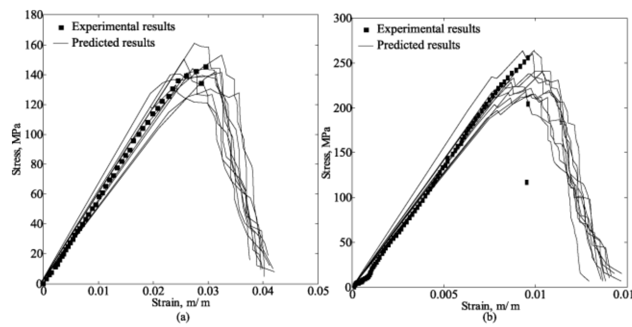

Fig. 5. Stress-strain response of bulk specimens obtained from experiments for (a) inner specimen with 18 fibre bundles and (b) outer specimen with 34 fibre bundles. Corresponding numerical simulations for inner and outer specimens are also plotted over the experimental results. Note that, numerical results are obtained through 10 random realisations.

Discussion

The exercise of fitting experimental data reveals the fact that a classical two-parameter Weibull probability distribution represents the ultimate strain $\varepsilon_{\text{i}}^{\text{u}}$ and corresponding cross-section areas of fibre bundles A_i adequately. Also, the tensile strength $\sigma_{\text{i}}^{\text{u}}$ is well represented with modified Weibull distribution. Looking at the accuracy and repeatability of the results in the material like bamboo, this technique seems to be reliable. However, the results obtained here, especially the shape and scale parameters α and β, are not of use as a reference for describing the properties of any other natural or synthetic fibres bundles. The fact that the shape and scale parameters α and β used in this work are obtained from experiments on one particular bamboo species restricts its applicability to other materials.

Nevertheless, Weibull statistics is a good way to describe the properties of the fibre bundles as the linear-elastic behaviour of many highly disperse bundles can be effectively generated with the shape and scale parameters found in this work. As the individual fibre bundle in bulk bamboo shows difference in properties and high dispersion when tested alone, the overall response of bulk bamboo is difficult to predict. However, the effect of dispersion in properties loses its effect as the number of fibre bundles in the specimen increases.

Conclusion

A local variety of bamboo is studied with a view to establish an implementable method of predicting the tensile strength of bulk specimen. A classical two-parameter Weibull statistics is used to fit experimental data of the tensile behaviour of fibre bundles. In this work, it is shown that

1. The cross section area and ultimate strain bamboo fibre bundles are well represented with the classical Weibull statistical function.
2. The tensile strength with respect to cross sectional area is represented on a modified Weibull statistics.
3. The estimates of tensile strength obtained using Weibull probability distribution have been tested against experiments on specimens drawn from the bulk bamboo.
4. The results obtained through numerical simulations closely match with the experimental results justifies the use of Weibull statistics for the present work.

Appendix A. Weibull parameters determination

Input arguments: Experimental data *x* arranged in ascending order, and number of data n

Output arguments: Weibull parameters α and β

```
for j = 1 to n do
% Probability for each data point
P(j) = (j − 0.5)/n
% Abscissa X for each data point
X(j) = ln x(j)
% Ordinate Y for each data point
Y(j) = ln(−ln (1 − P(j)))
end for
% Linear regression of Y = AX + B
(A, B) = LinReg(X(j), Y(j))
% Weibull parameters α and β
α = A
β = exp (−B/α)
```

Appendix B. Procedure of failure model for bulk bamboo

Input arguments: Weibull shape and scale parameters β_A, α_A, $\beta_\varepsilon, \alpha_\varepsilon, \beta_\sigma, \alpha_\sigma$ for number of data n

Output: Tensile behaviour of bulk bamboo for realisations *m*

```
% Randomise properties of each fibre bundle
for j = 1 to n do
A_i: wblrand(β_A, α_A)
ε_i^u: wblrand(β_ε, α_ε)
σ_i^u: wblrand(β_σ, α_σ)
E_i = σ_i^u / ε_i^u
end for
% Calculate overall cross-section area of the bulk specimen
A^(s) = Σ A_i
% Obtain volume fraction for each fibre bundle
for j = 1 to n do
k_i = A_i / A^(s)
end for
% Sort ultimate strain of the fibre bundles in ascending order
ε_1^u < ε_2^u < ⋯ < ε_n^u
% Determine the bulk specimen strength related to each fibre bundle failure
for j = 1 to n do
% Bulk specimen ultimate strain is equal to the ultimate strain of fibre bundle
ε_1^(s) = ε_i^u
%Secant modulus of the bulk specimen
E_(sec)i^(s) = Σ_{j=1}^n k_j * E_j
Determine the strength of bulk bamboo
σ_i^(s) = E_(sec)i^(s) * ε_i^(s)
% Set rigidity of fibre bundle i to zero at break
E_i = 0
end for
```

Keywords

Natural fibres, bamboo, constitutive relation, statistical methods, Weibull probability.

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