Fiber-reinforced Cementitious Composite: Sensitivity Analysis and Parameter Identification

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Methods and software tools used to identify the material parameters of high-performance cementitious composites are presented. The aim is to provide techniques for the advanced assessment of the mechanical fracture properties of these materials, and the subsequent numerical simulation of components/structures made from them. The paper describes the development of computational and material models utilized for efficient material parameter determination with regards to a studied composite. This determination is performed with the help of experimental data from four-point bending tests. The data is used in inverse analysis based on artificial neural networks. Sensitivity analysis plays an important role in the process. It is a part of a complex methodology for the statistical and reliability analysis of structures made of high-performance cementitious composites. The procedure also utilizes statistical simulation of the Monte Carlo type for the preparation of a training set for the artificial neural network utilized in the material parameter identification process. In the case of fiber-reinforced concrete, the simulation mainly includes tensile strength, modulus of elasticity and the parameters of the tensile softening model.

Introduction

The advanced assessment of mechanical fracture properties is of primary importance for subsequent numerical simulations of components/structures made of fiber-reinforced concrete (FRC). A key aspect when performing nonlinear fracture mechanics modeling is certainly the available knowledge about the tensile softening model and its parameters, as well as the corresponding fracture energy which dissipates during the cracking process, as was emphasized, e.g. in [**1, 2**].

The variability of experimental results obtained using specimens made of quasi-brittle materials such as FRC is relatively high due to the natural heterogeneity of the material. This means that the assessment of its mechanical fracture parameters is much more difficult and problematic than would otherwise be so. It is not sufficient to merely remain at the deterministic level when designing or assessing structures made of this material, and indeed this can even be risky. Therefore, it is highly recommended that the statistical assessment of experimental measurements be employed together with stochastic nonlinear simulation and a probabilistic approach to structural design.

The aim of the performed research is to deepen existing knowledge about the behavior of the studied FRC material especially in relation to its resistance to crack propagation. The obtained knowledge is a prerequisite for the efficient design and nonlinear computational simulation of this composite and the subsequent expansion of its applicability in order to increase the sustainability of constructed elements, structures and buildings.

The research consists of several parts: First, a suitable constitutive law for FRC cementitious composites was developed within ATENA nonlinear fracture mechanics software [**3**]. The computational model was verified using experimental data provided by the Malaysian company DURA Technology Sdn Bhd. Second, sensitivity analysis was performed. It showed the importance of individual material model parameters related to the response of specimens tested under laboratory conditions in the four-point bending test configuration. With the help of the sensitivity analysis results, software for the identification of material parameters was developed. This program, which is called FRCID-4PB, implements the inverse analysis method based on an artificial neural network (ANN) [**4, 5**] in combination with efficient statistical simulation [**6**].

The last step was the verification of the software and the implemented neural network by means of the comparison of identified material model parameters, and also via the comparison of structural responses obtained for both the original and identified input parameters. The proposed methodology and software are based on experimental and computational methods falling within the field of fracture mechanics, soft computing and reliability theory.

There are several recent applications of ANN for fiber-reinforced composites or general engineered cementitious materials. For example, in [**7**], selected properties of engineered cementitious composites were predicted using an ANN which had been trained using data collected from the literature. Another practical example can be found in [**8**], where an ANN was employed for the prediction of tensile and compressive strengths at 28 days of hardening. The ANN was trained using experimental data for hundreds of different mixtures. In the above-mentioned references, the ANNs are utilized as regression models for the prediction of properties based on experimental data. On the other hand, the utilization of the ANN in this paper is quite different, and is uniquely for cementitious composites. Here, selected material parameters of FRC are identified based on inverse analysis and the virtual simulation of a fracture test. The ANN is used here as a surrogate model describing the inverse relationship between structural response parameters and material parameters.

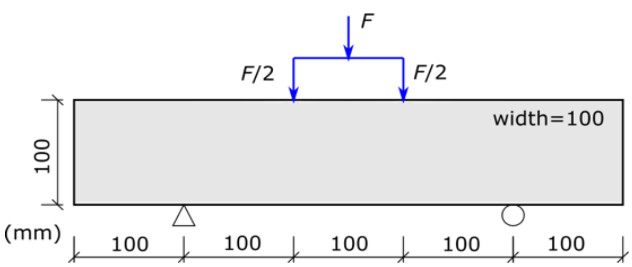

Fig. 1. Schematic view and dimensions of an unnotched specimen tested in the four-point bending configuration.

Computational model

Mechanical fracture parameter values are determined using the results of fracture tests in suitable test configurations. In the case of FRC, four-point bending (4PB) tests on unnotched prism specimens are widely used (**Fig. 1**). The outcome of each test is a force–deflection diagram (*F*–*d* diagram, **Fig. 2**), which is subsequently used as input data for mechanical fracture parameter identification (which is an inverse task). Nonlinear numerical simulations of such four-point bending beam tests were performed using ATENA FEM software [**3**], [**9**]. This software enables the response of the structural member to be calculated, including material damage.

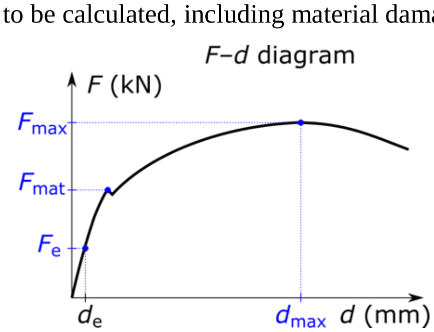

Fig. 2. A typical force vs. midspan deflection diagram obtained from a four-point bending test performed on a fiber-reinforced concrete specimen.

The constitutive law at each point in the material plays a crucial role in nonlinear numerical analysis. The realism of the structural response, damage and failure calculated by the computational model depends strongly on the quality of the material model, which determines the exactness and accuracy of the achieved results. Since FRC is a rather complex heterogeneous material with a strongly nonlinear response, the "3D Nonlinear Cementitious 2 User" material model was utilized for the realistic modeling of this composite. It can capture all the important aspects of the FRC material's behavior and response under tensile as well as compressive loading.

Concrete in tension is described within the smeared crack concept by the nonlinear fracture mechanics with crack band approach. The main material model parameters are tensile strength and the shape of the softening function, which characterizes the crack mouth opening in relation to the remaining tensile stress. In the model, a real discrete crack is represented as a band of localized strains where the strain corresponding to the crack width is related to the size, shape and orientation of the finite element. The softening function for the material law used in the smeared crack concept has to be determined for every point in the material (or finite element) in such a way that the objective crack opening law will be preserved. Only such an approach based on an energy-related formulation assures an objective solution independent of the finite element mesh [**3**].

In the case of fiber-reinforced composite, in order for a softening function to be appropriate it must reflect all the typical aspects of FRC cracking. The proposed function for the optimal reproduction of the tensile softening of the investigated composite material is shown in **Fig. 3**. The form of the function is described by tensile strength and four additional parameters, C_1 to C_4. Its form attempts to capture all the particular stages of damage to the FRC material, such as the reaching of ultimate strength by the cement matrix (see the point in **Fig. 3** where stress reaches f_t), the delayed activation of the fibers within the composite (see parameter C_1), and the consequent stiffening of the composite with activated fibers after the bearing capacity of the matrix is exhausted (see the stiffening part of the diagram until the strain reaches the C_2 value).

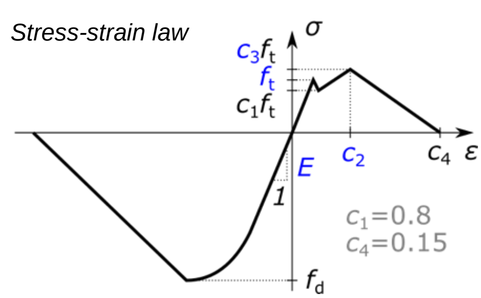

Fig. 3. Stress–strain law utilized for the fiber-reinforced concrete material.

Concrete in compression exhibits a strong confinement effect – i.e. an increase in the compressive strength under a concentrated three-dimensional compressive stress state. This effect is covered by the special plasticity theory with a non-associated plastic flow law in the combined fracture-plastic models in ATENA [**9**]. The compressive ductility of FRC should be appropriately accounted for in the material model as well.

Sensitivity analysis

A parametric study was performed in order to understand the effect of the individual material parameters on the beam response in the four-point bending test. The aim was to identify the parameters which can be termed dominant, along with those that do not influence the response and thus can be excluded from the inverse analysis. An additional aim was to determine the real ranges of the dominant parameters and propose a suitable set of response parameters that will be used as inputs for the inverse analysis.

The parametric study was performed step by step for six model parameters – tensile strength, compressive strength and four parameters of the tensile softening model, C_1 to C_4. The value of each parameter was gradually changed. This was always in four steps (with the exception of compressive strength, where only two steps were considered), while the remaining parameters always remained unchanged at their default values. One of the cases always corresponded to the default set of parameters – marked with the abbreviation "ini" in the figures below. The resulting *F*–*d* diagrams are shown in **Fig. 4**.

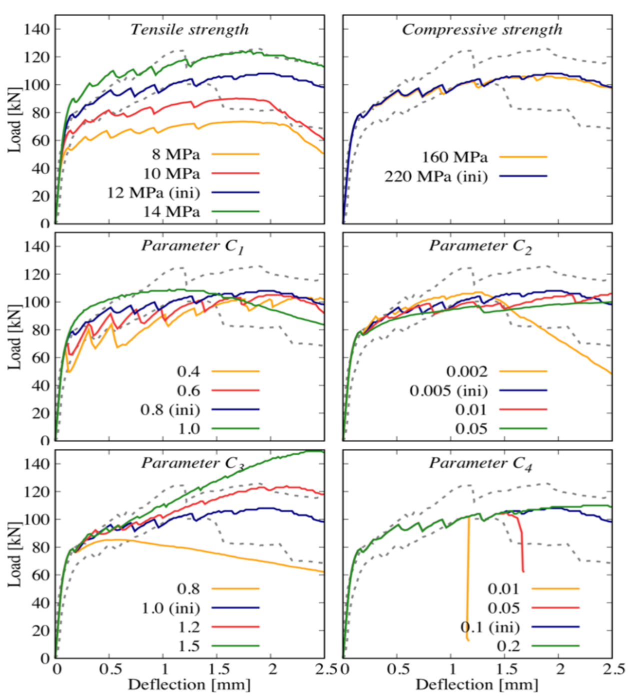

Fig. 4. Four-point bending test simulation for various values of tensile strength (top left), compressive strength (top right), parameter C_1 (middle left), parameter C_2 (middle right), parameter C_3 (bottom left), and parameter C_4 (bottom right).

The following conclusions can be drawn from the parametric study results:

- Tensile strength is one of the dominant parameters. Increasing its value leads to increased resistance across the entire load spectrum.
- Compressive strength, on the other hand, is one of the parameters that have no effect on the resulting response when the sample is loaded in four-point bending. Therefore, this parameter will not be included during identification and will be considered at its default value.
- Parameter C_1, which is related to the delayed activation of fibers in the cement matrix, is expected to have the greatest influence on the shape of the diagram when the cement matrix capacity is exhausted. Its size controls the "depth" of the first drop in the curve.
- Parameter C_2 does not affect the initial part of the diagram, but affects the amount of deformation when the ultimate bearing capacity of the specimen is reached.
- Parameter C_3 is also closely related to the ultimate bearing capacity of the sample, but this time it mainly influences the ultimate force and slope of the second part of the diagram, when it is strengthened due to the joint action of the matrix and the fibers.
- Parameter C_4 influences the maximum deformation that will be achieved during the test. When C_4 has a low value, the numerical test is terminated at a low deflection value.

For the purposes of inverse analysis, we can conclude:

- Due to the clear effect of parameter C_1 on the first visible decrease in the *F*–*d* diagram when reaching the ultimate bearing capacity of the cement matrix, it is possible to exclude it from the identification process and consider it to have a constant value of $C_1 = 0.8$. In the case of any future need to identify samples with a significantly “deeper” first drop in the diagram, parameter C_1 may be included in identification.
- Another parameter whose value can be considered constant and thus can be excluded from identification is C_4. Its initial value $C_4 = 0.1$ is large enough to obtain the required deformation, which corresponds to the ductility of the material.
- The last parameter that will not be employed in the identification process is compressive strength. Its effect on the bending response is negligible.
- The identification set thus contains four material model parameters – tensile strength, C_2 and C_3, and the modulus of elasticity, whose influence on the slope of the initial linear part of the diagram is well known.

Methodology and software implementation

An artificial intelligence-based inverse procedure developed by Novák and Lehký [**4**] is able to transform fracture test response data into a set of desired mechanical fracture parameters. This approach is based on matching laboratory measurements with the results gained by reproducing the same test numerically. The ANN is used here as a surrogate model of an unknown inverse function between the input mechanical fracture parameters **P** and the corresponding response parameters **R**:

$$\mathbf{P} = f_{\mathrm{ANN}}^{-1}(\mathbf{R}) \tag{1}$$

As described in the previous section, the identification set of parameters **P** contains four material model parameters – tensile strength, C_2 and C_3, and the modulus of elasticity. With respect to the results of the sensitivity analysis, the following four response parameters were selected for the response set **R**: force value F_e from the linear elastic part of the force vs. midspan deflection diagram, force value F_{mat} when reaching the ultimate bearing capacity of the cement matrix, force value F_{max} when reaching the ultimate bearing capacity of the whole specimen, and the corresponding deflection value d_{max}.

The cornerstone of inverse analysis is an artificial neural network, which is of the feed-forward multilayer type [**10**]. The most important step in the whole procedure is the creation of the network and its training – the adjustment of its synaptic weights and biases. The set for training the ANN is prepared numerically via the utilization of an FEM model using a stress-strain law, as shown in **Fig. 3**. It simulates 4PB testing with random realizations of material parameters. These are generated with the help of the stratified sampling method (LHS) and by performing an inverse transformation of the distribution function in order to reflect the probability distribution of the parameter.

The random responses obtained from the computational model and the corresponding random realizations of parameters serve as input–output elements of the ANN training set. After training, the ANN is ready to solve the main task, which is to provide the best material parameters in order for the numerical simulation to achieve the best agreement with the experiment. This is performed by simulating a network using the previously measured responses as an input, resulting in a set of identified material parameters. The last step is result verification – the calculation of the computational model using the identified parameters. Comparison with the experiment will show the extent to which the inverse analysis was successful. More theoretical backgrounds and a detailed description can be found in [**4**].

Note that the importance of training sample preparation was previously emphasized and tested by Tong and Liu [**11**], including the LHS scheme. In spite of the fact that these authors concluded that number-theoretic methods appear to be the most efficient, the LHS scheme also provided very good results. Moreover, our focus on LHS is also determined by the general applicability of this small-sample simulation technique for practical statistical, sensitivity and reliability analyses in many fields of engineering.

The above-described method of parameter identification, which combines nonlinear simulations with the training of an artificial neural network, is relatively time consuming and of high complexity. Therefore, the whole procedure was implemented in FraMePID-3PB software [**5**] and successfully used for the material parameter identification of plain concrete. The software has now been modified and updated for fiber-reinforced concrete, including the tensile softening model, as described in the section on the computational model. A screenshot of the first version of FRCID-4PB software is shown in **Fig. 5**.

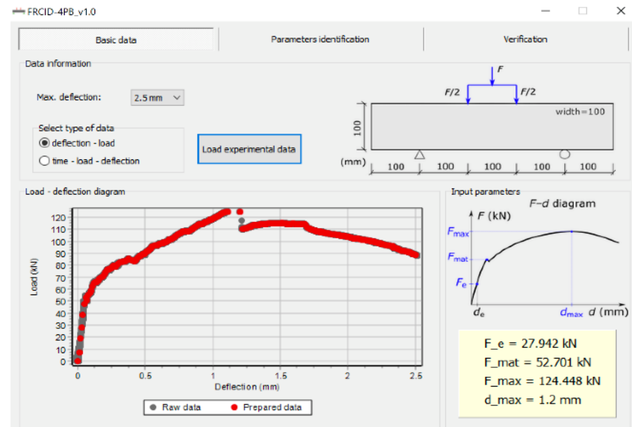

Fig. 5. FRCID-4PB software: loading of experimental data and evaluation of input parameters for identification.

Results of identification

Initial verification of the software and implemented neural network was performed using a set of five randomly simulated sample responses loaded in the four-point bending configuration. The structural response corresponds to the behavior of the Dura composite. Based on the sensitivity analysis presented above, four parameters of the material model were the subject of identification – modulus of elasticity, tensile strength, C_2, and C_3; see Fig. 3. Because the input set of material parameters was available, it was possible to compare their values directly with the values obtained via identification.

A graphical comparison of all five test samples is shown in **Fig. 6**. Furthermore, a comparison of the resulting sample responses in the form of force–displacement diagrams is depicted in **Fig. 7**. The results show very good agreement between the original and identified data. Parameter C_2 seems to provide the least accurate identification; its sensitivity is relatively low compared to that of other parameters. This discrepancy did not have any significant effect on the identified force–displacement diagrams, as shown in **Fig. 7**.

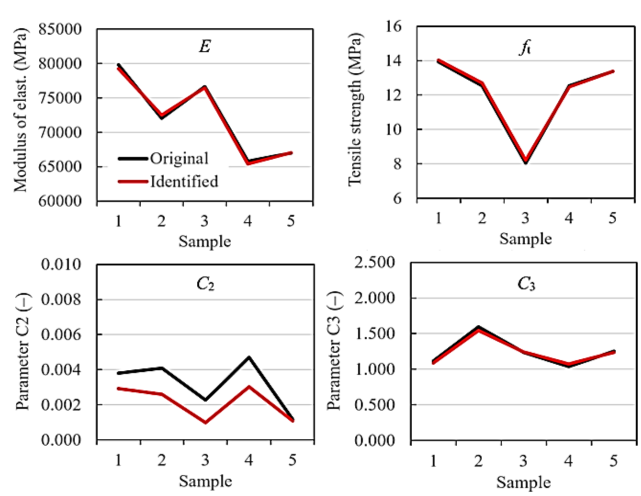

Fig. 6. Validation of identified parameters: original vs. identified material model parameters.

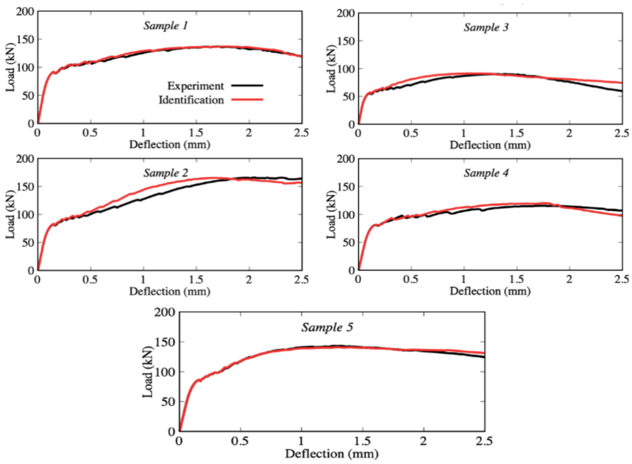

Fig. 7. Verification of identified parameters: comparison of original vs. simulated structural response for five selected samples.

When comparing experimental and numerical force–displacement diagrams for all tested specimens one can see a very good fit of curves along the whole loading path. In particular, the initial elastic parts, the first drops related to the delayed activation of fibers and the forces when reaching the ultimate bearing capacities of specimens are in perfect agreement. Just a small discrepancy can be seen for sample No. 2, where the numerical simulation exhibits a slightly higher stiffness for the composite with activated fibers compared to the experiment. This discrepancy is probably related to some numerical instability which occasionally occurs when performing global nonlinear analysis.

Conclusion

The paper describes a methodology and software tool which can be routinely used for the indirect determination of the fracture–mechanical parameters of fiber-reinforced concrete based on data recorded during four-point bending tests on unnotched prismatic specimens. The user-friendly software consists of a predefined and trained artificial neural network for the fast identification of material parameters. The whole concept is based on a combination of the statistical simulation method, finite element modeling and artificial neural networks.

The methodology and software tool were verified using data from the Malaysian technology company DURA Sdn Bhd. The main conclusions which can be drawn from the sensitivity analysis and subsequent identifications are:

- The proposed stress–strain law utilized for the FRC material reflects all the typical aspects of its cracking.
- The dominant material parameters for the description of material behavior when subjected to four-point bending are modulus of elasticity E, tensile strength of cement matrix f_t, and four parameters of the tensile softening model, C_1–C_4. For the tested DURA composite, parameters C_1 and C_4 can be adjusted directly from the experimental load vs. deflection diagram, while the remaining four parameters are the subject of identification.
- Note that the tensile strength of the cement matrix of fiber-reinforced composite is determined via inverse analysis too. Such separation is quite unique for mixtures such as FRC, which are composed of several constituents.
- With respect to the parameters to be identified, four suitable response parameters were selected as the input for inverse analysis.
- The presented validation results show outcomes from sensitivity analysis used as a supporting tool for the development and proper set-up of an artificial neural network and its components.
- The subsequent performance of parameter identification on tested FRC samples proved the ANN's efficiency and ability to identify material parameter values leading to the accurate simulation of the response of the studied composite.

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Author's contributions

Plan conceived by: DN, RP; Sensitivity study: DN, DL; Data analysis: DL, RP; Software development: DN, DL, RP; Identification: DL; Paper written by: DN, DL, RP. The authors have no competing financial interests.

Keywords

Fiber-reinforced cementitious composite, concrete, nonlinear modeling, sensitivity analysis.

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