Stress Fields at the Central Point of Arc Crack under Uniaxial Tension

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Abstract

In this study the stress fields under uniaxial tension of the plane with arc cut were calculated. To describe the concentration of normal and shear stresses in the areas of the crack ends, new characteristics based on equilibrium conditions were introduced. The influence of the angle of the crack arc opening and the direction of the plane tension was studied. It was found that those factors changed the value and the stresses sign at the crack ends. The opening angles of arc cracks and tension direction promoting the development of cracks under the influence of tensile strain, tensile stresses or combination of the stresses were determined. The promoting conditions that inhibit the cracks were studied.

Keywords: Stress field, uniaxial tension, arc, crack, tensile strain.

Introduction

One of the powerful and convenient methods to treat two-dimensional crack problems is the complex potential function method by Kolosov and Muskhelishvili [**1-3**]. Among various mathematical methods in plane elasticity, solving problems on the stress field of cracks using Muskhelishvili methods [**1**] gives good impression about the properties of the pole cracks. Physically poles treated as hubs with an infinitely high voltage. Then it is assumed that the characteristics of the pole completely determine the production of the crack, and therefore take the limit to short distances from the crack tip (asymptotic approximation), this ideology is discussed on literature work [**2, 3**]. However, the stronger materials as well as an infinitely high resistance degradation that would not crack the poles constrained. In the theory of equilibrium for cracks [**4, 5**] the gap between the cracks in the top gradually decreases. It is assumed that these forces cancel tearing stress on the pole so that the stress at the crack tip becomes finite. In the general formulas for the stress, this situation is realized by removing the polar term.

A method for producing solutions for linear elastic problem areas shift without poles at the ends of the plot was developed [**6**]. It was shown that the method at [**6**] can be used for the shear fracture [**7**]. In [**8**] the plane arc crack revealed a strong field with a high concentration of tensile stresses, is not related to the ends of the crack. Thus, the failure of the assumption of the existence of stoppers with an infinitely high resistance to fracture. Some key factors are, the possibility of making elastic problems without poles, the existence of significant hubs of tensile stresses in areas not connected to the ends of the cracks that indicated the asymptotic approximation gives enough correct and complete description. For a complete description of cracks approximation should be abandoned.

This article shows the results of the stress field at the ends of the arc crack in an elastic plane with a uniform field under infinity stretching. The problem of the stress field of the arc crack considered earlier [**12, 13**], but only for the crack opening angle equal to 180 degree, and in the asymptotic approximation.

The Mathematical Method

Homogeneous elastic plane with a circular arc (**Fig. 1**) is subjected to a uniaxial tensile homogeneous at infinity. It is assumed that edges of the cut are free from stress, the crack between the two sides make a gap, which is assumed to be small so that its effect on the stress field can be neglected and it is not large enough so that the deformation of the plane does not reduce surface cut at contact.

Elastic problem of the extension of the plane with an arc cut, whose edges are stress-free, was solved in [**9-12**] to the problem of conjugation information. Complex potentials, which were used to calculate the characteristics of the field, are listed below:

$$U(z) = \frac{1}{2X(z)}\left\{C_0 z + C_1 + \frac{D_1}{z} + \frac{D_2}{z^2}\right\} + \frac{D_0}{2} + \frac{\overline{\Gamma}}{2z^2} \quad (1)$$

$$\Omega(z) = \frac{1}{2X(z)}\left\{C_0 z + C_1 + \frac{D_1}{z} + \frac{D_2}{z^2}\right\} - \frac{D_0}{2} - \frac{\overline{\Gamma'}}{2z^2} \quad (2)$$

where

$$X(z) = \sqrt{(z-a)(z-b)}; C_0 = \frac{1}{2}\left(\Gamma' - \overline{\Gamma'}\right).\sin^2\frac{\theta}{2} + \frac{4\Gamma + \left(\Gamma' + \overline{\Gamma'}\right).\sin^2\frac{\theta}{2}.\cos^2\frac{\theta}{2}}{2\left(1 + \sin^2\frac{\theta}{2}\right)};$$

$$C_1 = -C_0\cos\theta; D_1 = -\overline{\Gamma'}\cos\theta; D_2 = -\overline{\Gamma'}; and D_0 = 2\tilde{A} - \tilde{N}_0 \quad (3)$$

where z = x + iy is a point in the complex plane containing the arc with unit radius centered at the origin (Fig. 1), a and b are coordinates of the ends of the arc cut-out, θ – is half the opening angle of the arc, the values of Γ and Γ' are sets stress at infinity. The crack in the middle is subjected to a tensile load. As the material is stretched, elastic energy is stored. Crack propagation becomes thermodynamically favorable; the elastic energy released per crack extension is equal to the surface energy necessary to form the new surfaces. Fig.1. also shows a contour plot of the stress field, which diverges at the crack tip. Adjusting the stress, surface energy, and Young's modulus changes the critical stress, all units are arbitrary.
The uniaxial tensile stress with p:

$$\Gamma = p/4, \Gamma' = -(p/2)\exp(-2i\alpha) \quad (4)$$

where α is angle, measured counterclockwise from the axis of symmetry of the arc to the stretching direction. Secondary (auxiliary) functions are calculated from (1 and 2) as:

$$\Psi(z) = \frac{1}{z^2}U(z) - \frac{1}{z^2}\overline{\Omega\left(\frac{1}{\bar{z}}\right)} - \frac{1}{z}\frac{dU(z)}{dz} \quad (5)$$

$$N(z) = \mathrm{U}(z) + \Omega\left(\frac{1}{\bar{z}}\right) + \bar{z}\left(\bar{z} \quad \frac{1}{z}\right)\overline{\Psi(z)} \quad (6)$$

$$\delta(z) = \frac{iz}{2\mu}[(3-4\nu)U(z) + U(\frac{1}{z}) - z(z - \frac{1}{z})\psi(z) \quad (7)$$

The value of the hydrostatic pressure:

$$P(z) = 4\frac{1+\nu}{3}\mathrm{Re}[U(z)] \quad (8)$$

where, ν is the Poisson's ratio (assumed to be 0.3). The stress tensor components can be given as:

$$\sigma_{rr}(z) = \mathrm{Re}[N(z)]; \sigma_{\beta\beta}(z) = P(z) - \sigma_{rr}(z); \quad (9)$$
$$\tau_{r\beta}(z) = \mathrm{Im}[N(z)]$$

The derivatives of the components of the displacement vector can be written as:

$$\frac{\partial u_x}{\partial \chi} = \mathrm{Re}[\delta(z)], \frac{\partial u_y}{\partial \chi} = \mathrm{Im}[\delta(z)] \quad (10)$$

where χ =arg(z). Asymptotic transition was not performed.

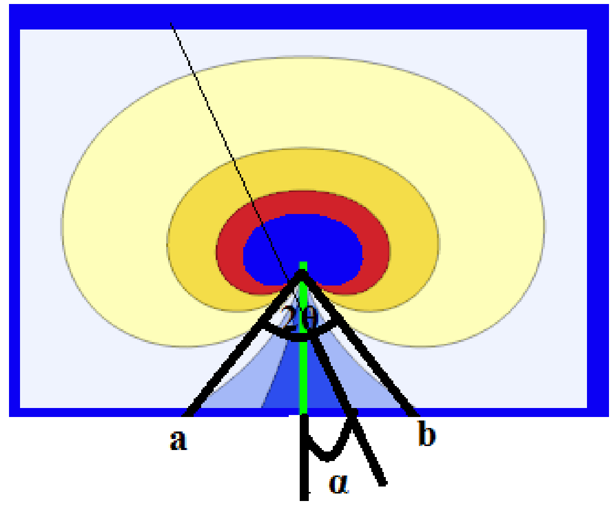

Fig. 1. Layout arc crack and acting load with a contour plot of the stress field, which diverges at the crack tip.

The calculation of the stress field changed the opening angle of the crack (2θ) from 20° to 180° with increments of 5-10°. The angle of the direction of stretching (α) changes from 0° to 90° in increments of 5-10°. Radial angle (β) was measured from the vertical axis counterclockwise.

Results and discussion

The calculation results of the stress tensor distribution of perpendicular and tangent to the arc of unit radius are shown in **Fig. 2**, and **Fig. 3**. These data indicate that the shear stress at the endpoints of the arc crack is not constant. The intensity and value of stress change depending on the two angles 2θ and α. For example, on the left end of the crack in **Fig. 2a** there is a pole, which was tearing at high stress. At the other end also has a terminal, but it is defined as a compressive strain. In **Fig. 2b**, the right end of the pole is no crack. Similar results were obtained for the tangent of the stresses (**Fig. 3**).

The magnitude of the stresses themselves is not convenient for quantifying the stresses in the end sections of cracks, since these stresses can be infinite. Therefore, other characteristics were adopted.

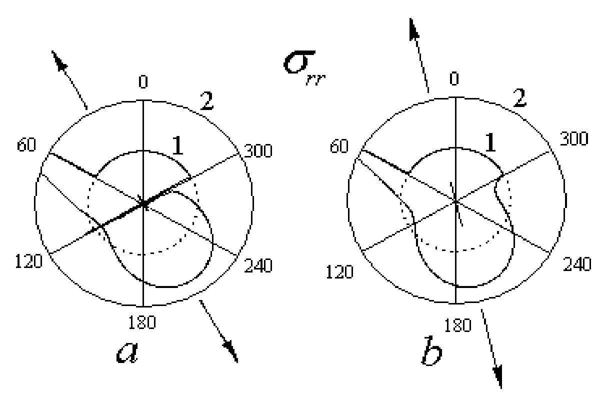

Fig. 2. The distribution of σ_{rr} along an arc of unit radius for the angle 120° of an arc crack under uniaxial tension at angles 30° (a) and 16° (b).

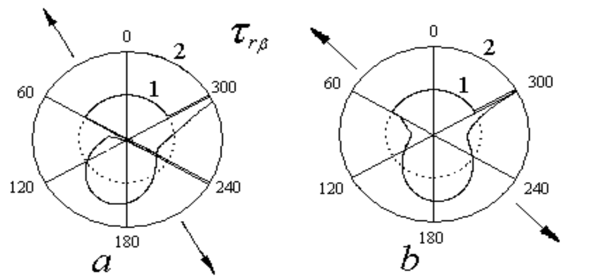

Fig. 3. The distribution of $\tau_{r\beta}$ along an arc of unit radius for the angle 120° of an arc crack under uniaxial tension at angles 30° (a) and b = 48° (b).

Static stress fields satisfy the conditions of equilibrium: the sum of the amount of mechanical forces and moments acting on any area of the plane are zero. Consider a circle of unit radius centered at the origin, on the circumference of the circle creates external tensile stress.

$$\sigma_{rr}(\beta) = \sigma_{\infty} \cdot [0{,}5 \cdot \cos[2 \cdot (\beta + \pi - \alpha)] - 0{,}5] \tag{11}$$

$$\tau_{\kappa\beta}(\beta) = \tau_{\infty} \cdot [0{,}5 \cdot \sin[2 \cdot (\beta + \pi / 2 - \alpha)]] \tag{12}$$

The crack is located along the arc of the circle. Where, the free edges of the crack, the crack length at the normal to the line of fracture and shear stresses are absent. Equilibrium in equations (11 and 12) is valid. Then the rest of the boundary circle of the unit circle should have stresses that reached equilibrium. These stresses are related to the field of a crack. The solution of the elastic problem gives stresses representing to the sum of the external field and the stress field associated to the crack. Then the components of the force that restores the equilibrium can be written as:

$$F_x = \int_0^{2\pi} [\sigma_{rr}(1,\beta) - \sigma_{\infty}(\beta)] \sin\beta d\beta,$$

$$F_y = \int_0^{2\pi} [\sigma_{rr}(1,\beta) - \sigma_{\infty}(\beta)] \cos\beta d\beta. \tag{13}$$

where sin β and cos β are the projecting factors. The integrations are determined by the intensity of the terminal poles. Therefore, as a quantitative measure of the intensity of the poles at the ends of the crack with respect to the normal component of the stress tensor which can be written as:

$$J_a = \int_a^{\pi} [\sigma_{rr}(1,\beta) - \sigma_{\infty}(\beta)] d\beta,$$

$$J_b = \int_b^{-\pi} [\sigma_{rr}(1,\beta) - \sigma_{\infty}(\beta)] d\beta \tag{14}$$

For the right and left ends of the cracks, respectively. As measures of the intensity of the poles along the tangent component, the moments of forces acting on the boundary of the unit circle were used, and are shown as:

$$M_{\grave{a}} = \int_{\hat{a}}^{\pi} \left[\tau_{r\beta}(1,\beta) - \tau_{\infty}(\beta)\right] d\beta,$$

$$M_b = \int_b^{-\pi} \left[\tau_{r\beta}(1,\beta) - \tau_{r\beta}(\beta)\right] d\beta \tag{15}$$

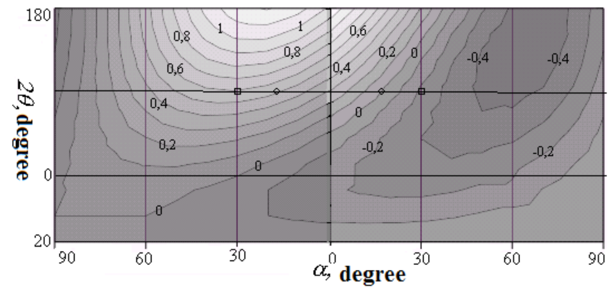

Fig. 4. The characteristic Intensity distribution of the normal stresses to the arc which acting at the ends of the arc crack, at 2θ - α. The left half of the figure refers to the left end of the crack and the right to the right.

The same for the right and left ends of cracks, respectively. **Fig. 4** and **Fig. 5** show the distribution of the intensity characteristics at the field poles 2θ-α. The data obtained show the combination of loading parameters, contributing to the development of cracks. Therefore, the most dangerous, less dangerous, and those in which the edges of the crack are closed, are prerequisites for inhibiting the cracks. They are highly dangerous because of the normal stresses acting at α ≈ 30° and the relatively large angles of cracks on the left end of the fracture (**Fig. 4**). At the right end at α ≈ 60° represent compressive stresses, with such stresses, cracks do not develop.

Under the influence of shear crack, shear stresses (**Fig. 5**) will predominantly develop at the right end at α ≈ 15° with large opening angles. Which are the highest stresses acting for cracks with 2θ ≈ 130°. At the left end of the cracks with small α, the shearing stresses act in the opposite direction to it takes place at the right end. When α is close to 45° and the middle corners of the opening angles of the arcs shearing stresses crosses zero, and change sign, but then remain relatively small.

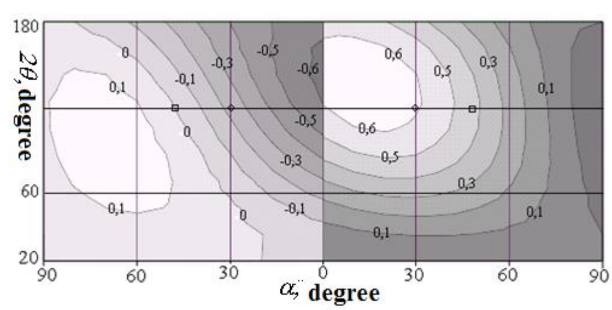

Fig. 5. The distribution of the mechanical torques which acting along a semicircle of unit radius at 2θ - α. The left half of the figure refers to the left end of the crack and the right to the right.

The lines with zero values of normal and shear stresses have a different path. Near 2θ ≈ 60° and α ≈ 30° at the left end they intersect. The crack does not develop in the region of the angle fields adjacent to this combination. With other combinations, the development of a crack can be determined by the action of stresses or by their cooperative action. From the perspective of the joint action of normal and shear stresses, the most unsafe region for the right end of the crack at 2θ ≈ 150° and α ≈ 15°.

Conclusion

The end concentrations of cracks having the shape of a circular arc depend on the crack opening angle and the direction of the external stretching field. Both tearing and shearing stresses at the ends vary in magnitude and sign. Calculations of the characteristics of these stresses made it possible to identify combinations of conditions that contribute and inhibit the development of arc cracks.

Scientific and practical value of the work is that the results can be used for the development of the theory of elastic deformation in the study of materials, also can be used for the development of the modern understanding of the processes occurring at the micro/nano level in solids. The results can be used as demonstration material for students of material science, for the possible creation of a laboratory practical.

Author's contributions

Authors have no competing financial interests.

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