Brief Review: Simulation of Novel Systems Using Duality Quantum Algorithm

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Abstract

In this article, we review the recent works of quantum simulation of novel systems briefly, the parity-time-reversal-symmetric (PT-symmetric) quantum system and the Yang-Baxter-equation (YBE) system, using duality quantum algorithm. Duality quantum algorithm studies the linear combinations of unitary operators, making it possible to simulate non-unitary evolutions of novel quantum systems. A PT-symmetric quantum system is a typical non-Hermitian system of which the evolution is not unitary and cannot be simulated directly by a conventional quantum computer. A recent work by C. Zheng has established a theory to simulate a general PT-symmetric two level system by duality quantum computing. The other typical example is the YBE quantum systems, of which the evolutions can be both unitary and non-unitary. C. Zheng and S. J. Wei described a theory that the two hand sides of the YBE can be simulated efficiently by the duality quantum algorithm in their recent research. Perspectives of future applications are expected at last. Copyright © VBRI Press.

Keywords: Quantum simulation, duality quantum algorithm, PT-symmetry, Yang-Baxter equation.

Introduction

Quantum information science is one of the hottest frontier areas nowadays. One of the aims is to produce practical quantum computers. On one hand, it is to meet the technological problem that quantum effects would appear as the increasing of the density of integrated electronic circuits. On the other hand, it is from the science itself as Feynman gave an enlightening idea that quantum physics can be simulated by nature itself [**1**], opening a way to investigate quantum systems efficiently. Therefore, quantum simulation becomes the one of the motivations and is still a main research area of the quantum information science.

Scientists have made a lot of efforts from different areas, and quantum computers are becoming practical technologies especially for small physical systems. For instance, many low dimensional quantum systems can be investigated by quantum simulators [**2**-**14**]. To simulate a quantum system, it starts from constructing an effective Hamiltonian. The time evolution of the system can then be simulated and realized. Because a conventional quantum computer is charged by the law of standard quantum mechanics, only the Hermitian quantum system can be simulated directly. The standard quantum mechanics requires a Hermitian Hamiltonian, and thus the evolution of the system is unitary. For non-Hermitian systems, they cannot be simulated directly by a conventional quantum computer in principle in that the time-evolutionary operators are not unitary. Beside conventional systems, many novel quantum systems have novel properties, attracting attentions of scientists increasingly. For example, a parity-time-reversal quantum system has many novel phenomena and properties that are valuable for investigations and applications, e.g., the quantum entanglements in the system, quantum brachistochrone problem, and etc. However, the non-Hermitian systems evolve non-unitarily, and a conventional quantum computer cannot achieve the evolutions directly. It is demanded methods that can simulate non-Hermitian quantum systems.

In this review article, we will base on two recent research works (see references [**15**] and [**16**]) which are related to the parity-time-reversal-symmetric quantum system and the Yang-Baxter-equation physical system, respectively. Evolutions of both the two systems are non-unitary which cannot be simulated in a conventional quantum computer directly. Duality quantum algorithm using the linear combination of the unitary operators is hired to construct the related non-Hermitian subsystems in a higher dimensional Hilbert space, and the evolutions of the subsystem will evolve as the non-unitary operators.

Duality quantum algorithm

Duality quantum algorithm is first proposed in 2002 [**17**] and is being developed after that time [**18**-**24**]. It investigates and utilizes the linear combinations of unitary operators, enabling non-unitary evolutions of novel quantum systems to be simulated in conventional quantum computers. Duality quantum algorithm has been applied in many tasks [**25-29**], e.g. efficient quantum simulation of open system [**25, 26**], arbitrary two-qubit processing in solid quantum system [**27, 28**], test of quantum foundmental theory [**29**], and etc.

To realize the linear combination of operators, e.g., two terms, an ancillary qubit and another subsystem to be investigated are needed. The schematic scheme of a duality quantum algorithm with a combination of two terms is shown in **Fig. 1**. It would achieve the sumation of the two unitary operators of U_3 and U_4 acting on the work subsystem with the ancillary qubit being on some quantum state. The quantum state of the whole system would be prepare in a pure state in general, and the initial quantum state of the ancillary qubit would be prepared in a logical state $|0\rangle_a$. Passed through the circuit in **Fig. 1**, the initial quantum state would be in an output state

$$c_1|0\rangle_a(U_3+U_4)|\psi\rangle_s + c_2|1\rangle_a U|\psi\rangle_s \quad (1)$$

where, the parameters c_1 and c_2 are non-zero numbers, and U is a unitary operator that is not focused here. Finally, we measure the whole system. If the ancillary qubit is on state $|0\rangle_a$, i.e.,

$$\frac{1}{k}|0\Big\rangle_a (U_3+U_4)|\psi\rangle_s \quad (2)$$

where, k is equal to the module of the sumation of the two unitary operators. Then the sumation of the two unitary operators would be achieved.

Following are brief review of two examples based on our recent works [**15**, **16**] that can illustrate how to use duality quantum algorithm to simulate novel quantum systems. The first one is an example to simulate a general PT-symmetric two-level quantum system. The other investigates the quantum simulation of the Yang-Baxter equation efficiently.

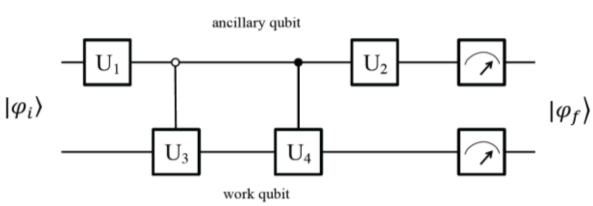

Fig. 1. Quantum circuit for schematic illustration of the duality quantum algorithm realizing two-term combinations. In the quantum circuit, it contains an ancillary qubit and a work subsystem that would evolve as the novel system to be simulated. The initial quantum state of the whole system would be prepared in a pure state. After it passes through the quantum circuit, the sum of the two terms, i.e., the two unitary operators U_3 and U_4, would be achieved and act on the work subsystem conditionally based on the ancillary qubit.

Example I: PT-symmetric quantum system

For the conventional quantum mechanics, the requirement that a Hamiltonian should be Hermitian, $H^+ = H$, is to keep the physical system observable, i.e., the eigenvalues of the Hamiltonian should be real numbers. However, the Hermitian requirement is not the necessary condition that eigenvalues are real. Another class of Hamiltonians, which satisfy the relation $PH^+P = H$, are also meet the observable need that the eigenvalues are real. The relations that these Hamiltonians satisfied are called parity-time-reversal-symmetric symmetry, which are investigated by Prof. C. M. Bender in 1998 [**30**]. After that time, the theory and applications of PT-symmetric quantum mechanics are studied and developed heavily [**31-44**], attracting plenty of scientists' attentions.

The evolution of a quantum system is decided by the Hamiltoian. For a Hermitian quantum system, the evolution operation is unitary because of the requirement of the Hermitian symmetry, which leads to the conjugate is the same as the Hamiltonian itself. The conventional quantum computers in the physical world are Hermitian systems, so they can simulate Hermitian system directly. For a PT-symmetric quantum system, however, the evolution is not unitary. Therefore, a PT-symmetric quantum system cannot be simulated in a conventional quantum computer directly.

In the recent work [**15**], we gave a proposal to simulate a general PT-symmetric two-level quantum system using duality quantum algorithm. Therefore, the ability of a quantum computer is further extended to simulate novel quantum systems and investigate the interesting properties such as quantum entanglements, quantum brachistochrone problems, and etc. We also studied the experimental realizations with a quantum optics system and an NMR quantum system, discussing their practicality.

Example II: Yang-Baxter-equation system

The Yang-Baxter equation was first introduced to solve the repulsive δ interaction problem in one-dimensional N particles [**45, 46**], and problems of statistical models on lattices [**47**]. A plenty of physical meaning of the YBE is revealed, and many links to a variety of areas of physics, such as statistical mechanics, group theories, and quantum field theory [**48-50**]. Therefore, the YBE has become a significant theoretical tool in physics today. In recent year, it is found gradually that there are natural links between the YBE and one of the hottest frontier research, quantum information and computing [**51-64**], that the braiding operators in the YBE are universal quantum gates and quantum entanglement has close relationship with the YBE. Having attracted lots of attention, the YBE is being investigated in quantum entanglement, quantum correlation, and topological quantum computing intensively.

In the recent work [**16**], it is proposed first time to simulate the Yang-Baxter equation using duality quantum algorithm. We construct a subsystem of the PT-symmetric system as a subspace of a Hilbert space with higher dimensions. Thus, it can be realized both in a conventional quantum computer and in a duality quantum compute. This proposal of duality quantum simulation keeps the YBE as a whole, i.e., simulating the two hand sides of the YBE simultaneously. It is needed to substitute the two unitary operators U_3 and U_4 by the operators of the two hand sides of the YBE. Therefore, the entanglements between the two hand sides of the YBE can be investigated. The whole system consists of the YBE subsystem and an ancillary qubit, and the operators can be substituted by the related forms. The simplest case if for a reduced two-dimensional Yang-Baxter equation, and a general case of the YBE were also studied.

Conclusion and future perspectives

Duality quantum algorithm can be used to simulate novel quantum systems, especially for non-Hermitian quantum systems, of which the evolutions are not unitary. Any novel phenomena and non-unitary evolutions in PT-symmetric two-level systems can be investigated using duality quantum algorithm in future. Non-unitary YBE can also be investigated and simulated using duality quantum algorithm in future.

Novel physical systems such as parity-time-reversal quantum has many novel phenomena and properties that are valuable for investigations and applications, e.g., the quantum entanglements in the system, quantum brachistochrone problem etc. The optimization of the evolutionary time between two quantum states can have applications in quantum computing. The Yang-Baxter-equation quantum system is full of physical meaning relating to quantum entanglements, topological quantum computing, and etc. Since the YBE is a whole equation, the two hand sides of it should be simulated simultaneously. Thus, the correlations and quantum entanglements of the two hand sides of the YBE are enabled be investigated efficiently in future.

Another future scope is that it is possible to investigate combinations of different non-Hermitian quantum systems. For example, we intend to find a YBE with PT-symmetry and simulate the non-unitary operators in the equation. On one hand, it will extend both the theories of PT-symmetric quantum mechanics and the Yang-Baxter equation. On the other hand, it would be a new application of the duality quantum algorithm, showing the strong ability of quantum computers.

In all, we believe more and more novel quantum systems can be investigate and simulated using duality quantum algorithm, and more novel phenomena will be revealed.

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