Bi-criteria optimization in designing magnetic composites

Krzysztof Z. Sokalski[1*], Barbara Slusarek[2], Marek Przybylski[2]

[1]*Computer Mathematics, Institute of Computer Science, Czestochowa University of Technology, Al. Armii Krajowej 17, 42-201 Czestochowa, Poland*
[2]*Magnets, Tele and Radio Research Institute, Ratuszowa 11 Street, 03-450 Warszawa, Poland*

[*]Corresponding author, Tel: (+48) 603122897, E-mail: ksokalski76@gmail.com

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Abstract

A novel algorithm for designing the values of the technological parameters for the production of soft magnetic composites (SMCs) was created. These parameters are the hardening temperature T and the compaction pressure p. These parameters enable us to optimize the power losses and magnetic induction. The advantage of the presented algorithm lies in bi-criteria optimization. The crucial role played by the presented algorithm is scaling, pseudo equation of state and fixed point. On this basis, mathematical models of power losses and magnetic induction were created. The model parameters were calculated on the basis of the power loss characteristics and hysteresis loops. The created optimization system was applied to specimens of Somaloy500. The obtained output consists of a finite set of feasible solutions. To select a unique solution, an additional criterion was formulated. Copyright © 2017 VBRI Press.

Keywords: Soft magnetic composites, power losses, magnetic induction, bi-criteria optimization.

Introduction

Soft magnetic composites (SMCs) have physical properties that are used for adapting these materials to specific applications [**1**], [**2**]. Highly often, the functionality of these materials depends on more than one feature. This dependence leads to multi-criteria optimization problems, which has not been applied yet in the design of SMCs. However, there are papers that treat more than one physical property of SMCs, but these are not considered to be target functions in an optimization procedure [**3**], [**4**].

Recently, an algorithm for designing the values of the hardening temperature and the compaction pressure in the production process of soft magnetic composites (SMCs) has been derived using the concept of the pseudo-state equation [**5**]. In equilibrium thermodynamics, the equation of state describes the relationships among the thermodynamic parameters. For example, in the case of a gas-liquid system, the temperature, pressure and volume of the considered material are the relevant parameters. By analogy with the equation of state, we consider a phenomenological relationship between the technological parameters and physical properties of the material. Such an approach for SMCs is possible due to the topology of the completed set of scaled power loss characteristics.

Topology of characteristics space

The most important features of characteristics topology are the following [**5**]. The set of characteristics consists of one variable smooth functions,

$$\frac{P_{tot}}{(B_m)^{\beta}} = F\left(\frac{f}{B_m^{\alpha}}\right), \quad (1)$$

where P_{tot} is the density of power loss, B_m is the peak of magnetic induction, f is the frequency of the electromagnetic field wave, and $F(\cdot)$ is a function of the following form [**5**],

$$F\left(\frac{f}{B_m^{\alpha}}\right) = \frac{f}{B_m^{\alpha}}\cdot\left(\Gamma_1 + \frac{f}{B_m^{\alpha}}\cdot\left(\Gamma_2 + \frac{f}{B_m^{\alpha}}\cdot\left(\Gamma_3 + \frac{f}{B_m^{\alpha}}\cdot\Gamma_4\right)\right)\right) \quad (2)$$

where, $\Gamma_{i,}$ α and β must be determined from experimental data. The form (1) has been derived from the assumption regarding the power losses as a homogeneous function in a general sense. Each characteristic is determined by the values of Γ_i coefficients and both the α and β exponents. These characteristics are functions of the technological parameters T and p by the following relations,

$$\Gamma_i = \Gamma_i(T,p),\ \alpha = \alpha(T,p),\ \beta = \beta(T,p), \quad (3)$$

where, T and p are hardening temperature and compaction pressure, respectively (1)-(3) reveal that characteristics of the samples composed at different T and p conditions possess different dimensions, whereas all the disentangled characteristics,

$$P_{tot} = (B_m)^{\beta}\, F\left(\frac{f}{B_m^{\alpha}}\right) \quad (4)$$

possess a common physical dimension. Why do we use the implicit form (1)-(2). Note that the right-hand side of this equation depends only on one effective variable $\frac{f}{B_m^{\alpha}}$. Therefore, calculations performed with (1) and (2) are represented by one curve for all values of f and B_m, whereas the results of the calculations performed with (4) are split into many curves. For example, if one requires P_{tot} as a function of *f,* then the number of generated characteristics is equal to the required number of different values of B_m.

Repultion between entangled characteristics

Let $Ch(\alpha_1, \beta_1, \Gamma_i^1)$ and $Ch(\alpha_2, \beta_2, \Gamma_i^2)$ be the two characteristics of the (1)-(2) form, where i=1,2,3,4. If $\alpha_1 \neq \alpha_2$ or $\beta_1 \neq \beta_2$ then these characteristics possess different dimensions. In general case for randomly selected values of the parameters $\alpha_1, \beta_1, \Gamma_i^1$ and $\alpha_2, \beta_2, \Gamma_i^2$ may correspond to intersecting characteristics, see **Fig. 2**. Intersecting points are singular due to different dimensions of the intersecting characteristics and they would reject the created here formalism. However, if we apply measurement data for estimation of the parameters values, then the all characteristics intersect only at origin point $\frac{f}{B_m^{\alpha}}$=0, for which the dimension is not very important. If we cut off this point then we obtain the space of repealing characteristics. Moreover all of the characteristics are monotonically increasing functions of $\frac{f}{B_m^{\alpha}}$.

According to the Egenhofer theorem [**6**] the relations between characteristics are invariant with respect to scaling, translation and rotation. Just the conservation of the relations with respect to the scaling enables us to use the implicit form of characteristics. According to (3) the power loss characteristics are parameterized by pressure and temperature. This dependence enables us to introduce a measure of distance in the considered space of characteristics. Let (T_1, p_1) and (T_2, p_2) be labels of the characteristics of the two composites that have been composed under conditions corresponding to these pressures and temperatures, respectively. In this case, the distance between these characteristics has the following general form:

$$\rho(T_1, p_1; T_2, p_2) = \sqrt{(\tau_2 - \tau_1)^2 + (\pi_2 - \pi_1)^2}, \qquad (5)$$

where, $\tau_i = \frac{T}{T_c}$ and $\pi_i = \frac{p}{p_c}$ are dimensionless temperatures and pressures, respectively. Where T_c and p_c are scaling parameters. Both T_c and p_c belong to optimization parameters' set. For the corresponding values of T_c and p_c see **Table 2**.

Therefore, the set of all characteristics constitutes the metric space. The introduced metric (5) has enabled us to ring-fence the two compact sets [**5**]. Each compact set corresponds to a physical phase that is defined by characteristic values of the physical parameters. For example, in [**5**], we considered the low and high-loss phases of SOMALOY500.

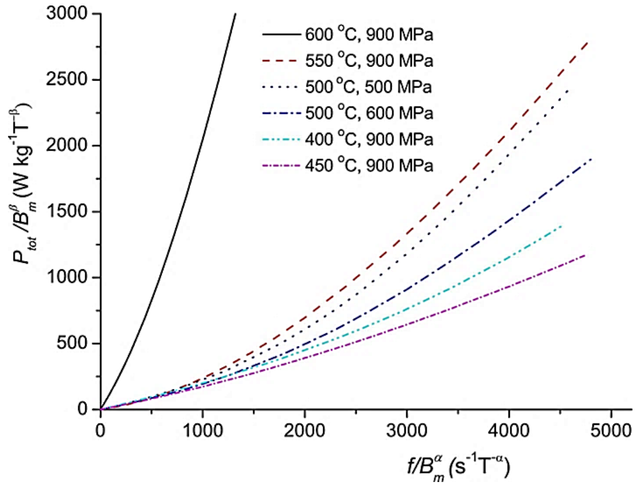

Fig. 1. System obeying scaling and Self-similar one. Five loss characteristics for *T<500* ℃ corresponding to the low-loss phase and the one characteristic for *T=600* ℃ corresponding to the high-loss phase.

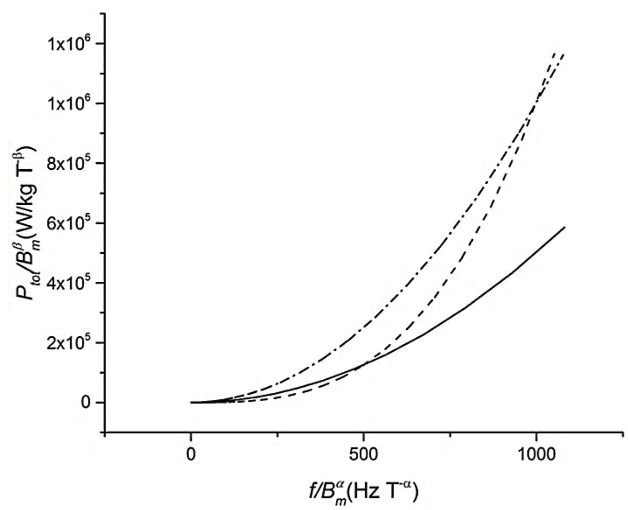

Fig. 2. System obeying scaling, however not Self-similar one. Example of not physical losses characteristics obtained for randomly chosen values of the model parameters.

All of the properties mentioned above are visible in **Fig. 1** and they enable us to introduce a measure of power loss$V(T,p)$, which is the average of the characteristics with respect to $\frac{f}{B_m^{\alpha}}$ [**5**]:

$$V(T,p) = \frac{1}{\varphi_{max} - \varphi_{min}} \int_{\varphi_{min}}^{\varphi_{max}} \frac{P_{tot}}{B_m^{\beta}} \, d\left(\frac{f}{B_m^{\alpha}}\right) \qquad (6)$$

Note that the dimension of the denominator in front of the integral and the dimension of integration limits cancel themselves out.

During the last decade, we have collected many experimental data confirming the revealed properties [**7**]. Basing on these data we conclude that repulsion of characteristics of scaled losses in soft magnetic materials as well as in soft magnetic composites is a low of the nature. This low injures the topological structure of scaled characteristics set which enable us to construct T,p dependent losses' measure (6).

Experimental data

Specimens were produced by cold pressing under pressures of 500…900 (MPa). The specimens made of SOMALOY500 powder were cured at a temperature of 400…600(℃) for 30 minutes in air atmosphere.

The specimens used in the experiments were ring-shaped with a square cross-section. The specimens had the following dimensions: external diameter = 55 (mm), internal diameter = 45 (mm) and thickness = 5 (mm).

The total power loss density P_{tot}, expressed in watts per kilogram (W/ kg), was obtained from measurements of the AC hysteresis cycle according to IEC Standard 60404-6 using the system AMH-20K-HS produced by Laboratorio Elettrofisico Walker LDJ Scientific. The total power losses P_{tot} were measured at a maximum flux density of B_m = 0,1...1,3 (T) over a frequency range of 10 to 5000 (Hz). During the measurements of the total power losses P_{tot}, the shape factor of the secondary voltage was equal to 1,111$\pm$1.5 %. The maximum measurement error of the total power losses was equal to 3%.

To optimize the magnetic properties, the magnetic inductions B at fixed magnetic field H equal to 1000 (A/m) were determined. These values were obtained from measurements of the DC magnetization curve according to IEC Standard 60404-4 using the same measuring system.

Power losses and magnetic induction pseudo-equations of state

Optimization of the power losses was based on the topological properties of the characteristics. However, in the case of the magnetic properties, the situation is considerably simpler. For optimization of the magnetic properties we selected magnetic induction B_{1000} for the fixed magnetic field $H = 1000(\frac{\text{A}}{\text{m}})$. As we have mentioned, we chose this value because the magnetic permeability of the soft magnetic composites reaches a maximum value at approximately this magnetic field.

We expected that the pseudo-equation of state would properly describe magnetic induction at this point as a function of T and p. In the previous paper [**8**], it was assumed and confirmed that the loss measure V obeys the scaling. Here, this assumption was extended to magnetic induction. To justify this assumption, we referred to two phenomena: invariance of the power losses (area of the hysteresis loop) with respect to scaling and invariance of the hysteresis loop with respect to scaling [**6**]. Therefore, for the bi-criteria optimization problem, minimization of the power losses and maximization of the magnetic induction for a fixed magnetic field, we used the following pseudo-equation of state of general form:

$$\mathrm{V(T,p)} = \left(\frac{p}{p_c}\right)^{\gamma} \cdot \Phi(X); \tag{7}$$

$$B_{1000}(T,p) = \left(\frac{p}{p_c'}\right)^{\gamma'} \Lambda(X'). \tag{8}$$

where, $\Phi(\cdot)$ and $\Lambda(\cdot)$ are arbitrary functions to be determined, and

$$X = \frac{\frac{T}{T_c}}{\left(\frac{p}{p_c}\right)^{\delta}}; \tag{9}$$

$$X' = \frac{\frac{T}{T_c'}}{\left(\frac{p}{p_c'}\right)^{\delta'}} \ . \tag{10}$$

where, $\gamma, \gamma', T_{c'}, p_c' \delta, \delta'$ are free parameters to be determined.

In the case of the power losses' pseudo-state equation, all calculations concerning the modeling of $\Phi(\cdot)$ and the fitting of the scaling exponents as well as model parameters, were performed in [**5**]. The most important result was the derivation of an infinite set of solutions for the technological parameters that minimized the power losses,

$$\frac{\frac{T}{T_c}}{\left(\frac{p}{p_c}\right)} = 19{,}75. \tag{11}$$

Pseudo-equation of state for magnetic induction B_{1000} will constitute a function of the two variables T and p. This function and the power losses' pseudo-equation of state (7) will enable us to optimize magnetic induction and losses together. The optimization criteria are the following: find $V = V_{min}$ and $B_{1000} = B_{1000,max}$ with respect to T and p.

Using the form for $\Phi(\cdot)$ [**5**] and the distance function (5), we reveal two phases of Somaloy 500: low loss and high loss ones. Therefore, in terms of the magnetic induction pseudo-equation of state, we must consider this phase separation. The measurement data of B_{1000} vs. T and p are separated into two subsets which correspond to revealed phases, see **Table 1**.

Table 1. Measure of magnetic induction B_1000 vs. hardening temperature and compaction pressure and Measure of power losses vs. hardening temperature and compaction pressure.

Measure of magnetic induction B_{1000} vs. hardening temperature and compaction pressure			**Power losses vs. hardening temperature and compaction pressure.**		
T (K)	p (MPa)	B_{1000} (T)	T (K)	p (MPa)	V $(\mathrm{W\,kg^{-1}T^{-\beta}})$
723,15	800	0,378	723,15	800	40,60
773,15	900	0,496	773,15	900	43,75
773,15	700	0,483	773,15	700	47,25
673,15	800	0,335	673,15	800	50,30
773,15	600	0,467	773,15	600	57,12
823,15	800	0,546	823,15	800	81,50
773,15	500	0,414	773,15	500	*89,28*
741,15	764	0,425	742,15	764	492,3
773,15	750	0,489	753,15	780	509,2
773,15	800	0,504	804,15	764	528,5
773,15	650	0,469	711,15	764	547,0
773,15	725	0,467	873,15	800	720,0
873,15	800	0,568	-	-	-

The horizontal line between $V = 89{,}28(\mathrm{W\,kg^{-1}T^{-\beta}})$ and $V = 492{,}3(\mathrm{W\,kg^{-1}T^{-\beta}})$ indicates the crossover between the low-loss phase and the high-loss phase. This transition is clearly visible in the jump of the $V(T,p)$ function around the separation line [**5**]. For each phase we assume an independent branch of the pseudo-state equation in the form of the Padè approximant. To simplify the notations, we apply abbreviations defined after formula (5). Expressing $\Lambda(\cdot)$ in (8) by the Padè approximant we derive the following form for the magnetic induction pseudo-equation of state,

$$B_{1000}(T,p) = (\pi')^{\gamma'} \frac{\tilde{G}_0+\tilde{G}_1X'+\tilde{G}_2X'^2+\tilde{G}_3X'^3+\tilde{G}_4X'^4}{1+\tilde{D}_1X'+\tilde{D}_2X'^2+\tilde{D}_3X'^3+\tilde{D}_4X'^4}. \quad (12)$$

where, $\tilde{G}_0, \ldots, \tilde{G}_4$, $\tilde{D}_1, \ldots, \tilde{D}_4$ are parameters of the Padè approximant **Table 2**. All parameters have to be determined with the data presented in **Table 1**. The corresponding pseudo-equation of state for the power losses has been derived in [**5**]:

$$V(T,p) = \pi^{\gamma} \frac{G_0+G_1X+G_2X^2+G_3X^3+G_4X^4}{1+D_1X+D_2X^2+D_3X^3+D_4X^4}. \quad (13)$$

Estimation of the parameters for the magnetic induction pseudo-equation of state

The above-mentioned crossover between the low-loss and high-loss phases is observed as a sudden change of V between two points: [773,15; 500,0] and [742,15; 764,0] in **Table 2**. However, this effect is not observed in the magnetic induction magnitude. Therefore, to have a compact description of the power losses and the magnetic induction, we take the crossover into account and we divide the data of **Table 2** into two subsets corresponding to the two respective phases. Minimization of χ^2 for both phases have been performed using Microsoft Excel 2010, where,

$$\chi^2 = \sum_{i=1}^{N} \left(B_{1000}(\tau_i', \pi_i') - (\pi')^{\gamma'} \frac{\tilde{G}_0+\tilde{G}_1X_i'+\tilde{G}_2X_i'^2+\tilde{G}_3X_i'^3+\tilde{G}_4X_i'^4}{1+\tilde{D}_1X_i'+\tilde{D}_2X_i'^2+\tilde{D}_3X_i'^3+\tilde{D}_4X_i'^4} \right)^2, \quad (14)$$

and where $N = 7$. **Table 2** presents estimated values of the model parameters for the low-loss phase. The pink sector of this table corresponds to the power loss' pseudo-equation of state, while the blue one corresponds to the pseudo-equation of state for magnetic induction.

Optimization of the magnetic induction and power losses

In the optimization of the power loss problem [**5**], we applied low-loss phase solutions, and high-loss phase solutions were not considered. However, it is not clear whether this simplification excludes important solutions for the magnetic induction. The binary relations are invariant with respect to scaling [7], [**5**]. This property enables us to present all the scaled characteristics in one picture **Fig. 3,** which allows us to draw the following conclusion. All the considered pressure characteristics of the high-loss phase are covered by the set of the low- loss phase characteristics. Therefore, for further investigations, we limit our searching to the low-loss phase. To this end, we draw part of the phase diagram of Somali 500 corresponding to the low-loss phase in **Fig. 4** and we deliver values of G_i, D_i, T_c, p_c, and γ, which are displayed in **Table 2**.

Table 2. Coefficients and exponents of pseudo-equations of state for the power losses and for the magnetic induction, pink and blue sectors of the table, respectively.

γ	δ	T_c	p_c	G_0	G_1	G_2
1,2812	0,1715	21,622	37,729	370315315	-47752251	1734952
G_3	G_4	D_1	D_2	D_3	D_4	-
-1,3764	-678,26	170,80	6243,8	386,96	-28,699	-
γ'	δ'	T_c'	p_c'	$\tilde{G}_0$	$\tilde{G}_1$	$\tilde{G}_2$
1,114	0,499	32,186	25,849	-784,41	764,05	-276,06
$\tilde{G}_3$	$\tilde{G}_4$	$\tilde{D}_1$	$\tilde{D}_2$	$\tilde{D}_3$	$\tilde{D}_4$	-
43,412	-2,431	3,649	2,901	2,837	3,975	-

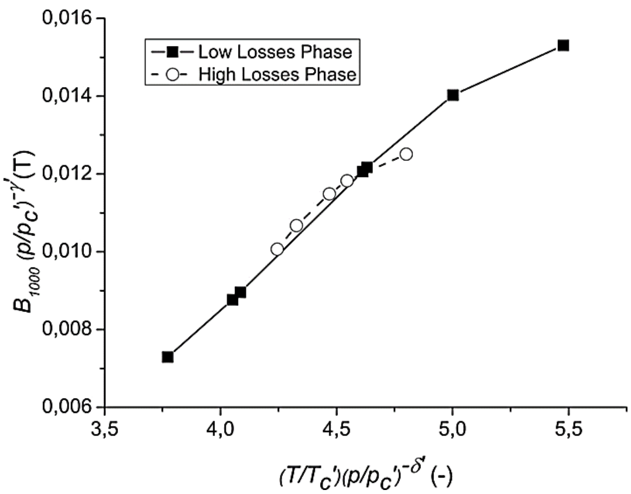

Fig. 3. Scaled B_{1000} vs. scaled temperature in the low- loss and high loss-phases.

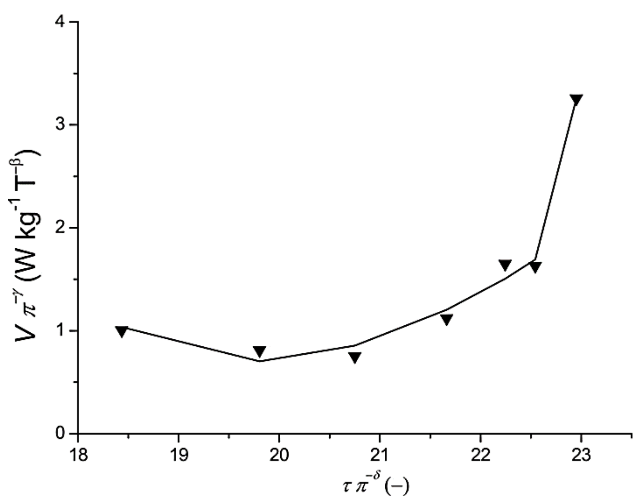

Fig. 4. Scaled V vs. scaled temperature in the low-losses phase. According to (9) $\tau\,\pi^{-\delta} = X$.

All calculations in this section must satisfy the following conditions: $18{,}4 < X < 22{,}9$, which based on the limitation of the presented calculations to the low-losses phase presented in **Fig. 4**.

The considered bi-criteria problem is formulated by the following items: initial value of $V = V_0$, the feasible set of (T,p) and the two criteria $V(T,p) = V_{min}$ and $B_{1000}(T,p) = B_{1000,max}$. Since increase of B_{1000} causes increase of V, these conditions are in contradiction. Therefore, criterion of the solution should be self-consistency between V_{min} and $B_{1000,max}$. Such consistency will be achieved as a fixed point of the following recurrence procedure.

Let the maximization and minimization procedures be represented by the operators $\hat{O}_{max}$ and $\hat{O}_{min}$, respectively. Let $V(T,p)$ and $B_{1000}(T,p)$ be functions defined by (12) and (13), respectively. In this case, the first step of optimization of $B_{1000}(T,p)$ and $V(T,p)$ can be written in the following form,

$$V(T,p) = V_0, \tag{15a}$$

$$\hat{O}_{max}B_{1000}(T,p) = B_{1000,max} \text{ for } T = T_1 \text{ and } p = p_1 \tag{15b}$$

$$B_{1000}(T,p) = B_{1000}(T_1,p_1) \tag{16a}$$

$$\hat{O}_{min}V(T,p) = V_{min} \text{ for } T = T_2 \text{ and } p = p_2. \tag{16b}$$

where, (15a) and (16a) play the role of the constraints for independent variables, while the (15b) and (16b) lead to the first two approximations of the solutions for T and p. Therefore, after n steps, we obtain,

$$V(T,p) = V(T_{2n},p_{2n}), \tag{17a}$$

$$\hat{O}_{max}B_{1000}(T,p) = B_{1000,max} \text{ for } T = T_{2n+1} \text{ and } p = p_{2n+1}, \tag{17b}$$

$$B_{1000}(T,p) = B_{1000}(T_{2n+1},p_{2n+1}), \tag{18a}$$

$$\hat{O}_{min}V(T,p) = V_{min} \text{ for } T = T_{2n+2} \text{ and } p = p_{2n+2}. \tag{18b}$$

Relations (15a)-(18b) generate the two converging series $T_1, T_2, \cdots T_{2n+2}$ and $p_1, p_2, \cdots p_{2n+2}$:

$$\lim_{i\to\infty} T_i = T^*, \lim_{i\to\infty} p_i = p^*. \tag{19}$$

where, (T^*,p^*) constitutes fixed point of (15a)-(18b) transformation. Substituting T^* and p^* to (12) and (13) we derive the meshed values of V and B_{1000}

$$V^*_{min} = V(T^*,p^*), \; B^*_{1000,max} = B_{1000}(T^*,p^*). \tag{20}$$

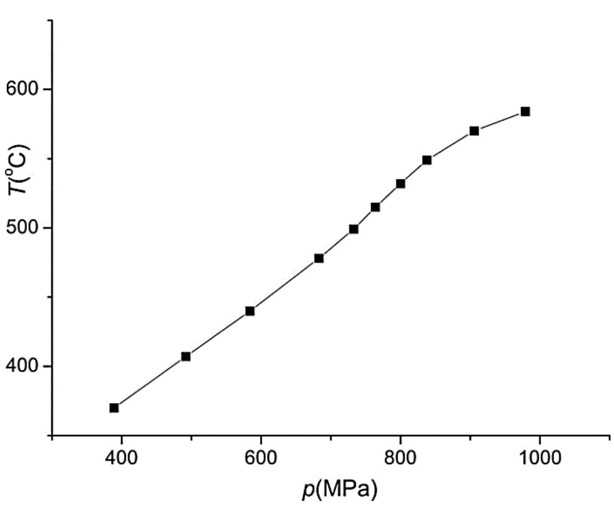

Fig. 5. Technological optimum curve presenting the dependence of the optimum temperature vs. the optimum pressure.

Results and discussion

Starting from different values of the initial point V_0 we have derived series of the fixed points. Optimization has been performed via the SOLVER routine of the EXCEL2010 program. **Fig. 5** and **Fig. 6** present these results in technological and in physical spaces, respectively. The obtained results represented by markers are fixed points of the proposed procedure. There is a one to-one correspondence between these points in physical and technological spaces.

To select a unique solution, one must provide an additional criterion resulting from a relation between the importance of losses and magnetic induction. For example, assuming the deepest minimum for the scaled measure of losses $V\pi^{-\gamma}$ **(Fig. 4)** we apply the condition given by (11). The intersection of two curves presented in **Fig. 7**, leads to the following single solution: $p = 382$ (MPa) and $T = 363$°C. In the physical space, this point corresponds to $V = 13{,}64(\mathrm{W\,kg^{-1}T^{-\beta}})$ and $B_{1000} = 0{,}29(\mathrm{T})$.

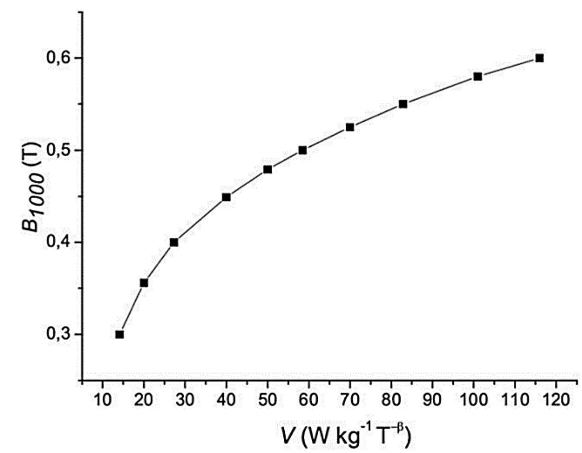

Fig. 6. Physical optimum curve. Optimum of magnetic induction B_{1000} vs. optimum of losses.

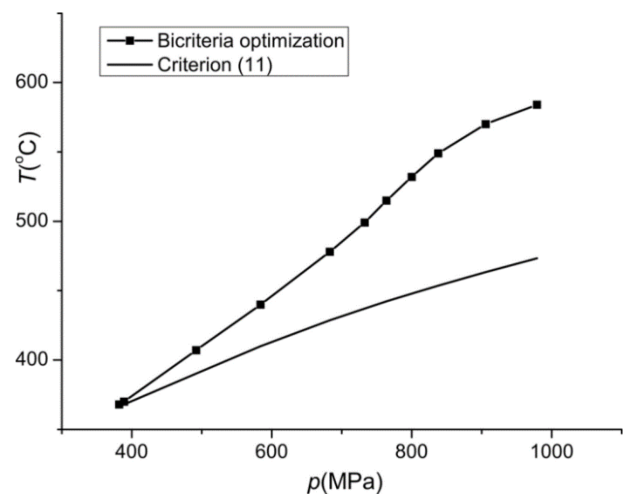

Fig. 7. Reduction of the feasible set of solutions for the technological parameters T, p to the single point $T = 368$ (°C), $p = 382$ (MPa).

Finally, we consider the power loss measure V. This measure is an auxiliary magnitude that helps us to derive values of designing technological parameters due to the following features:

(1) V is a pseudo-thermodynamic average with respect to the magnitude created with the peak of the magnetic induction and the frequency of the electromagnetic field wave. Therefore, V includes information about both independent variables.
(2) V depends on the technological parameters.

Conclusion

We presented novel method for the bi-criteria optimization of the chosen physical properties of Soft Magnetic Composites. Using this method, we solved the problem mentioned in [**5**], which concerns optimization of the losses and the magnetic induction. Achievement of the fixed point is interpreted as revelation of equilibrium between both assumed criteria. In the presented method the crucial roles are scaling, fixed point and the notion of pseudo-state equation. The created system is only as good as the experimental data, which are used for the estimations of the model parameters. Therefore, the first version presented here will be improved by forthcoming new experimental data. The presented example in this paper is a minimum nontrivial case of the Multiphysics problem and shows that this approach is suitable for designing Magnetic Composites. Therefore, the presented algorithm will be extended for more than two physical features of the composing material. For example, the design of magnetic composites also requires optimization of the mechanical properties because; the susceptibility of such materials to cracking in service is of fundamental concern [**9**]. During the last decade, we have collected many experimental data confirming the revealed properties [**6**]. Basing on these data we conclude in this paper, that repulsion of characteristics of scaled losses in soft magnetic materials as well as in soft magnetic composites is a low of the nature. This low injures the topological structure of scaled characteristics set which enable us to construct T,p dependent losses' measure (6). We target the derived algorithm to designers of SMCs.

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